Electrical Impedance Tomography to determine the location of moisture area of the wall

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Abstract— **The paper presents new method of determining the location of the moisture area of the wall using Electrical Impedance Tomography. To solve this problem the Boundary Element Method (BEM) was used to create the numerical model of the object under consideration. Using the synthetic data some results of the inverse problem solution was presented in this paper.**

I. Introduction

Capillarity ascending wall dampness is a serious problem in older buildings with no or insufficient waterproofing layers. It is dangerous for technical conditions of the walls and also for people's health. There are many different dehumidification systems. Monitoring the wall humidity during dehumidification proces is very important and it could be done using electrical methods, because conductivity is changing with humidity of the wall and its salinity. For example, the Electrical Impedance Tomography (EIT) technology may be used to determine conductivity distribution in the region under consideration. The method of boundary localization between dry and dump region of the wall by EIT is presented in this paper. The multi region Boundary Element Method was used to solve the forward problem of EIT in such case.

II. Boundary Element Method for Poisson equation in 2D space

The differential equation in two space dimensions can be written as follows:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial^2 x} + \frac{\partial^2\Phi}{\partial^2 y} = -b \qquad (1)$$

where Φ represents electric potential and x and y are the Cartesian coordinates. Now consider the physical domain of the problem. Figure 1 shows an arbitrary domain with some internal sources and boundary conditions, where the solution is sought.

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Let us assume that there is an interior point P (usually called the 'load' point) of coordinates x_P and y_P inside of the domain, and consider any point on the boundary Q (usually called the 'field' point) with coordinates x_Q and y_Q [2].

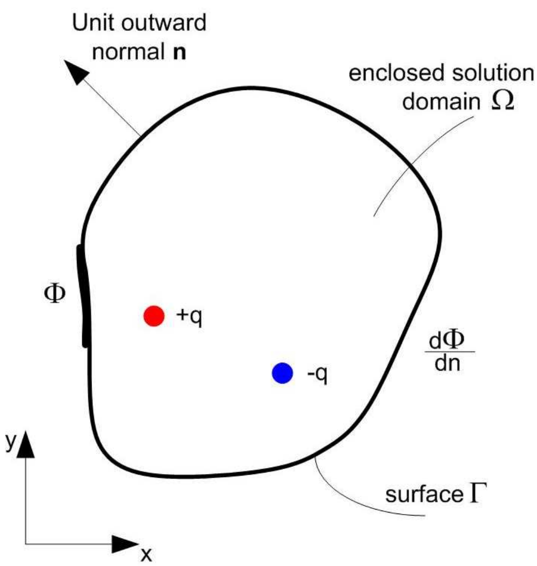

Fig. 1. Region under consideration with point sources and boundary conditions

The boundary integral equation equivalent to Eq. (1) is:

$$\frac{1}{2}u(\vec{r}) + \int_\Gamma \frac{\partial}{\partial n} G\left(\vec{r}, \vec{r}'\right) u(\vec{r}')d\Gamma = \int_\Gamma \frac{\partial u(\vec{r}')}{\partial n} G\left(\vec{r}, \vec{r}'\right) d\Gamma + \int_\Omega bG\left(\vec{r}_0, \vec{r}'\right) d\Omega \qquad (2)$$

where: $G\left(\vec{r}, \vec{r}'\right)$ - Green's function.

Current sources are modeled by the concentrated sources. They are a special case for which the function b at the internal point $\vec{r_0}$ becomes $q_0 = Q_0\delta(\vec{r_0})$, where Q_0 is the magnitude of the source and $\delta(\vec{r_0})$ is a Dirac delta function whose integral is equal to 1 over any volume containing the singularity point $\vec{r_0}$ and equal to zero elsewhere. Assuming that inside of the region we have n pairs of the sources one can write:

$$\frac{1}{2}u(\vec{r}) + \int_{\Gamma} \frac{\partial}{\partial n} G\left(\vec{r}, \vec{r}^{\,'}\right) u(\vec{r}^{\,'}) d\Gamma =$$
$$= \int_{\Gamma} \frac{\partial u(\vec{r}^{\,'})}{\partial n} G\left(\vec{r}, \vec{r}^{\,'}\right) d\Gamma + \tag{3}$$
$$+ \sum_{i=1}^{n} \left(q_{0_+,i} G\left(\vec{r}_{0_+,i}, \vec{r}^{\,'}\right) - q_{0_-,i} G\left(\vec{r}_{0_-,i}, \vec{r}^{\,'}\right) \right)$$

where $\vec{r}_{0_+}$ and $\vec{r}_{0_-}$ are the positions of the sources $+q_0$ and $-q_0$ respectively.

For 2D system, the fundamental solution for equation 1 is equal:

$$G\left(\vec{r}, \vec{r}^{\,'}\right) = \frac{1}{2\pi} ln \frac{1}{\left|\vec{r} - \vec{r}^{\,'}\right|} \tag{4}$$

where:

$$\left|\vec{r} - \vec{r}^{\,'}\right| = \sqrt{(x - x')^2 + (y - y')^2} \tag{5}$$

In order to solve the forward problem described by Eq. (3) the BEM was used and the boundary was discretized by zero order boundary elements [2], [3].

The matrix form of Eq. (3) is:

$$\mathbf{Au}(\mathbf{p}) = \mathbf{Bq}(\mathbf{p}) + \mathbf{Q}(\mathbf{p}) \tag{6}$$

The matrices **A** and **B** are non-symmetric and fully populated.

Fig. 2. Rising moisture created from the direct connection between soil and masonry.

III. Problem formulation

Rising moisture in masonry is very common in older houses without either horizontal or vertical waterproofing layers or drainages. In newer buildings, faulty construction is usually the cause for the moisture.

Rising moisture is created from the direct connection between soil and masonry. Porous natural stones or manufactured building bricks have, similar to a sponge, pores in which moisture is able to rise up against gravity. It can be seen in Fig. 2 [7].

Conductivity of the humid wall is much better than a dry one so it may possible to find boundary (interface) between humid and dry regions of the wall using EIT techniques.

In order to do this, it is necessary to place source and measure electrodes inside the wall. We used three source electrodes and eight measure ones, so we have 48 measurements: eight at each of six projection angles [6]. In Fig. 3 region under consideration, all electrodes and boundary conditions are presented. Conductivity of region 1 is higher than conductivity of region 2. The wall is 2 meters wide and 4 meters high. Three source electrodes are placed horizontally on height 0.4m at distances of: 0.5, 1 and 1.5m from left border of the region under consideration. Eight measuring electrodes are placed vertically at distance equal to 1 meter from left region border at heights: 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75 and 2m.

IV. Forward and inverse problem solution

The aim of the forward problem was to provide potential distribution inside the wall with exciting current sources. Changing position of current sources we have calculated voltage on measuring electrodes.

Simulation results for different positions of the boundary between regions 1 and 2 are presented in Fig. 4 - Fig. 7. The current sources and measuring electrodes positions are presented in Fig. 3. In all presented cases the conductivity of the region 1 is 5 times higher than the conductivity of the region 2.

The inverse problem was solved iteratively by repeatedly the forward problem solution [1], [4]. An objective function is defined as follows:

$$F(y) = \sum_{j=1}^{p} F_j(y) = \sum_{j=1}^{p} \left(\mathbf{f}_j - \mathbf{v}_{0j}\right)^T \left(\mathbf{f}_j - \mathbf{v}_{0j}\right) =$$
$$= \sum_{j=1}^{p} \sum_{i=1}^{n_{de}} \left(\mathbf{f}_{ji} - \mathbf{v}_{0ji}\right)^2 \tag{7}$$

where:

- j - projection angle (positions of energy source),
- $\mathbf{f}_j$ - vector of calculated potentials at measuring electrodes for j^{th} projection angle,
- $\mathbf{v}_{0j}$ - vector of measured potentials at measuring electrodes for j^{th} projection angle,
- n_{de} - number of collected measurement voltages.

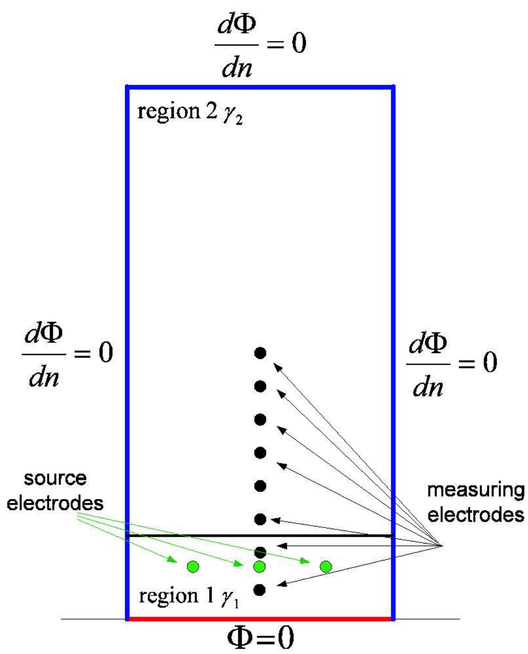

Fig. 3. Region under consideration.

In every iteration step was necessary to calculate objective function derivative. Derivative was calculated numerically and its central form was used in the inverse problem solution [5]:

$$\frac{dF}{dy} \approx \frac{F(y+\Delta y) - F(y-\Delta y)}{2\Delta y} \tag{8}$$

Calculation of derivative with the aid of Eq. (8) is more expensive regarding CPU time but it results in decreasing of solution error in comparison to formulae:

$$\frac{dF}{dy} \approx \frac{F(y+\Delta y) - F(y)}{\Delta y} \tag{9}$$

The stop criterion for iteration process was established as:

$$|yb_i - yb_{i-1}| < 1.e - 3m \tag{10}$$

where

- yb - boundary location,
- i - iteration step.

The inverse problem was solved for different cases regarding:

- the starting point,
- and conductivity ratio between regions $\frac{\gamma_2}{\gamma_1}$.

The history of the inverse problem solution with respect to number of iteration steps is shown in Fig. 9. The boundary localization errors for those cases are presented in Table I.

As one can see the boundary localization errors and number of iterations necessary to the inverse problem solution increase with decreasing ratio of the conductivities values between two regions.

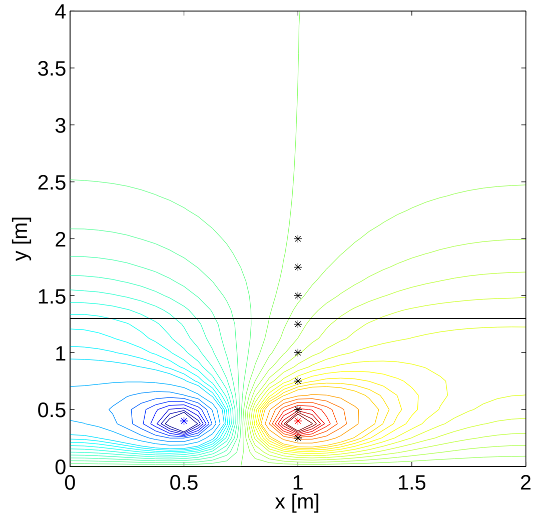

Fig. 4. Equipotential lines: boundary between regions at 1.3 m, sources in region 1, projection angle 1

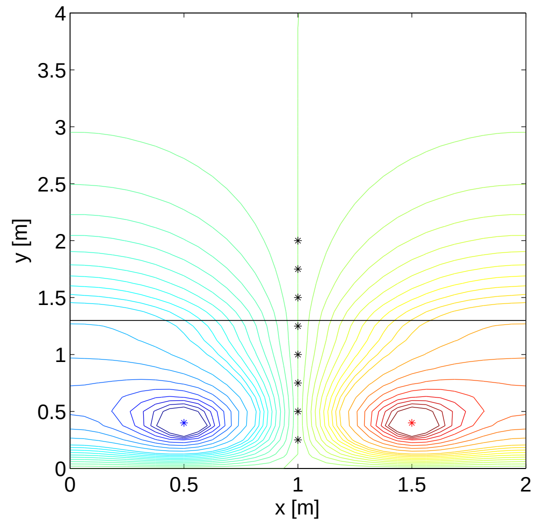

Fig. 5. Equipotential lines: boundary between regions at 1.3 m, sources in region 1, projection angle 2.

Table I. Boundary location errors for cases presented in Figure 9

case	**error [mm]**	**relative error [%]**
yb_1	1.16	0.09
yb_2	124.95	9.61
yb_3	0.16	0.01
yb_4	4.02	0.31

V. Conclusions

In this paper application of Electrical Impedance Tomography for determining the location of moisture area of the wall was presented. Boundary Element Method was used to solve the forward problem and then inverse problem was solved iteratively. Reported results prove that proposed algorithm is efficient and provide results with acceptable errors.

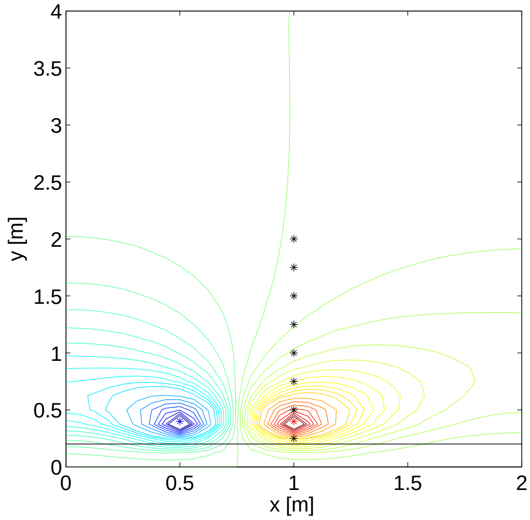

Fig. 6. Equipotential lines: boundary between regions at 0.2 m, sources in region 2, projection angle 1.

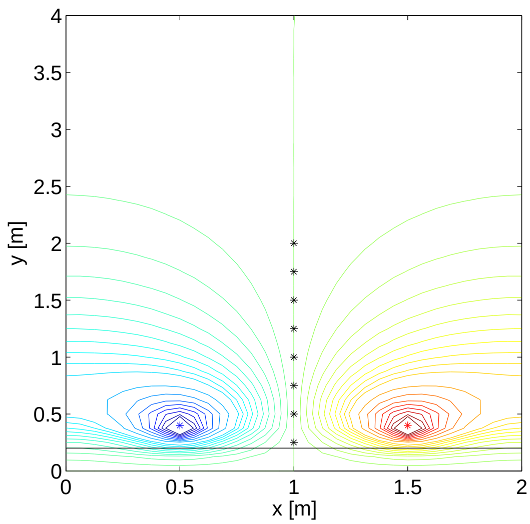

Fig. 7. Equipotential lines: boundary between regions at 0.2 m, sources in region 2, projection angle 2.

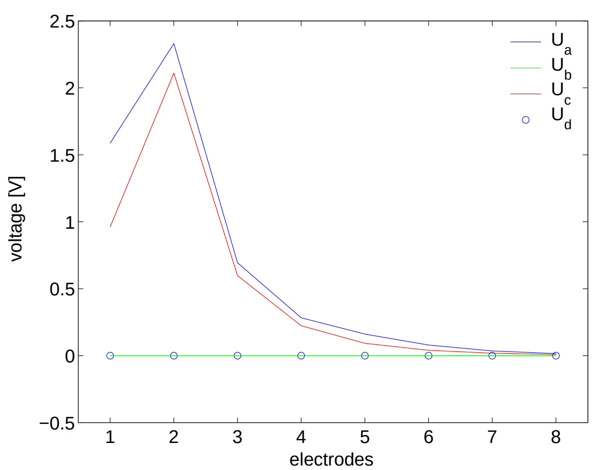

Fig. 8. Potentials on measuring electrodes for the cases presented on Fig. 4 (U_a), Fig. 5 (U_b), Fig. 6 (U_c), Fig. 7 (U_d).

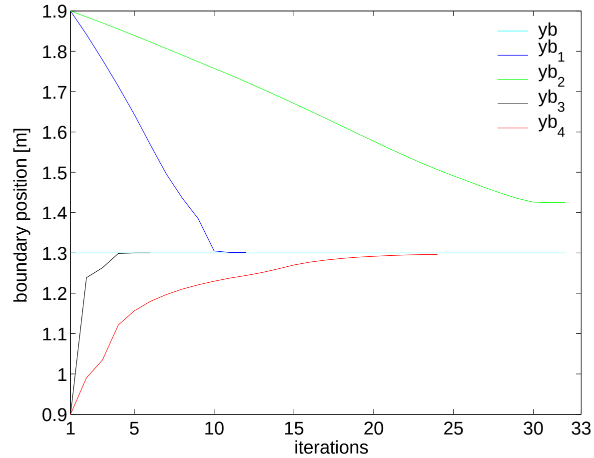

Fig. 9. Iterations of inverse problem for different starting points and different regions conductivities: $yb_1, yb_3 : \frac{\gamma_2}{\gamma_1} = 0.2$, $yb_2, yb_4 : \frac{\gamma_2}{\gamma_1} = 0.5$, where yb - desired position of boundary between regions 1 and 2.

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