On Subharmonic Oscillations

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***Abstract* — Conditions which promote the appearance of the second subharmonic are derived.**

The most general method to synthesize subharmonic oscillations of arbitrary waveform was proposed in [1]. Obtained differential equations are the third order and their structure is complicated. Differential equations [1] present most effective approach to solve the problem of synthesis of subharmonic oscillations.

In this brief some another problem is considered. We try to establish the conditions when subharmonic oscillations appear in the case when they are undesirable. From the practical point of view it is a very important problem for differential equation

$$\ddot{x} + f_1(x)\dot{x} + f(x) = F(t); \quad F(t+T) = F(t), \qquad (1)$$

which describes a lot of systems in electrical and radio engineering.

It is well known that in many papers where this problem was considered almost all results were obtained by numerical analysis or harmonic balance method. In consequence it is difficult to establish the structure of $f_1(x)$ which stipulates the possibility of occurrence of subharmonic oscillations.

Let us consider the second order differential equation

$$\frac{d^2x}{dt^2} + \varepsilon x \frac{d}{dt}\left[T_k(x) - \cos kt\right] + x = 0\,, \qquad (2)$$

where $T_k(x)$ is k-th Chebyshev Polynomial of the first kind, which is similar to the equation in variations for periodic solution (1). From the properties of Chebyshev Polynomial

$$T_k(\cos t) = \cos kt\,. \qquad (3)$$

For k=2 the equation (2) reduces to

$$\ddot{x} + 4\varepsilon x^2\dot{x} + (1 + 2\varepsilon\sin 2t)x = 0 \qquad (4)$$

it is obvious that $x = \cos t$ is periodic solution of (2)

The equation (2) is a mathematical model for subharmonic oscillations if and only if the solution $x = \cos t$ is unique steady-state.

For stability investigation let us introduce a new variable:

$$\xi = x - \cos t \qquad (5)$$

After linearization of (5) one obtains:

$$\begin{gathered}\ddot{\xi} + \varepsilon\cdot\cos t\cdot T_k'(\cos t)\cdot\dot{\xi} + \\ [1 - 0,5\varepsilon\cdot\sin 2t\cdot T_k''(\cos t)]\xi = 0\end{gathered} \qquad (6)$$

In partial cases k=2 and 3 it follows from (6)

$$k = 2 \quad \ddot{\xi} + 4\varepsilon\cdot\cos^2 t\cdot\dot{\xi} + [1 - 2\varepsilon\cdot\sin 2t]\xi = 0 \quad (7)$$

From (5) it follows that the most dangerous situation for appearance the second subharmonic arises when dissipative component is equal to $x^2\dot{x}$.

It is easy to prove that in accordance with Floque theory trivial solution $\xi = 0$ is stable. But it means only the stability in little (Liapunov stability). Numerical experiments confirm the global stability for ε< 4,25.

For k=3

$$\ddot{\xi} + 3\varepsilon\cdot\cos t(4\cos^2 t - 1)\dot{\xi} + [1 - 6\varepsilon(\sin t + \sin 3t)]\xi = 0$$

In this case dissipative component varies the sign and such model does not suit for explanation of appearance of the third subgarmonic in equation (1).

References

[1] Chua L., Green D. Synthesis of nonlinear periodic systems // IEEE Trans. on CT. 1974. Vol. CAS-21. NO.2. – Pp. 286-294.

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