An Analytical Model for Analysis of Magnetic Fields and Forces in Thin Toroidal System with Tilted Coils

Yu. Vasetsky, Yu. Rotachev

***Abstract* –The toroidal superconducting magnetic energy storage (SMES) systems and toroidal Tokamak fusion reactors resemble each other geometrically, where individual toroidal field (TF) magnetic coils are arranged around a circular axis. At the same time the high costs of support system materials stimulates to look for another configuration with smaller electromagnetic forces. The toroidal solenoid with tilted coils is one of such systems. In this paper an analytical model for analysis of magnetic fields and forces is developed. An ideal model with a thin toroidal current sheet, composed of an infinite number of current filaments, is considered. The configuration of each filament is the same as the configuration of the curvilinear axis of coil. For this condition a poloidal and toroidal components of surface density current were determined. The magnetic fields inside and outside circular torus were analyzed. The equations of linear density forces that act in tilted coils were obtained as a function of tilting angle.**

***Index term* – calculation model, magnetic fields, electrodynamic forces, magnetic energy storage.**

I. NTRODUCTION

Customarily, superconducting magnetic energy storage (SMES) systems as well as toroidal Tokamak fusion reactors have configuration, where individual toroidal field (TF) magnetic coils are arranged around a circle axis. These systems experience large radial and vertical forces, which require installation of substantial, expensive support structure [1-2]. Tilting of the TF coils from the vertical axis [3-4], however, produces several desirable effects on the magnetic fields and resultant forces. This introduces a poloidal component to the already present toroidal component of the magnetic field, and as a result electromagnetic forces of magnet coils are reduced. An example of the application of this concept on a large scale is a study that was performed to assess the impact of tilting the coils of the International Thermonuclear Experimental Reactor [5]. The coils were tilted $\pm 28^0$ and calculations showed that the associated reduction of radial forces is about 20%. The integrated vertical separating force, which acts on half of each coil, also decreased. Reduction of forces translates to a reduction of magnet support requirements, and, hence, a reduction of material costs.

In this paper a simple analytical model is presented to study magnetic fields and forces in thin toroidal systems with tilted coils then a ratio of small radius of torus to large one (the inverse aspect ratio) is small.

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II. MATHEMATICAL MODEL

An ideal model with a thin toroidal current sheet, composed of an infinite number of current filaments, is considered. The configuration of each filament is the same as the configuration of the curvilinear axis of a coil. This configuration is determined as the intersection line of the torus and the flat surface, as shown in Fig 1.

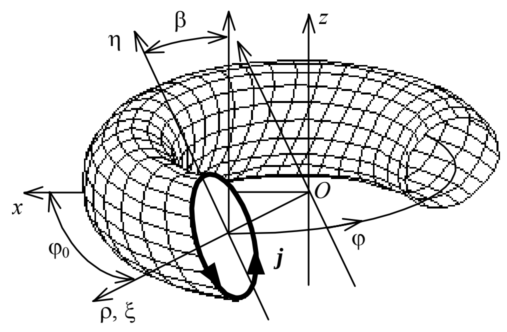


Fig. 1. Thin-shell torus composed of current sheet.

The configuration of a current line is determined by the following equations:

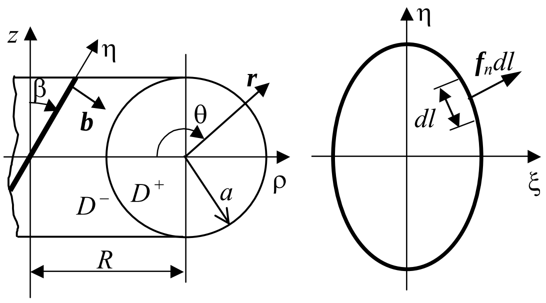
$$\begin{cases} \sin(\varphi - \varphi_0) = -\operatorname{tg}\beta \dfrac{z}{\rho}, \\ \rho = \rho(\chi), \; z = z(\chi). \end{cases} \tag{1}$$

The first expression is the equation of a flat surface that is turned around the radial axis on angle β; the following ones, in general case, are parametric equations that determine a surface of rotation about the z-axis.

The equations for a circular-section torus (Fig. 2a) may be written as

$$\rho(\theta) = R(1 - \varepsilon\cos\theta), \quad z(\theta) = R\varepsilon\sin\theta, \tag{2}$$

where $\chi = \theta$ is the angle shown in Fig. 2a, and $\varepsilon = \dfrac{a}{R}$ is the relative size of the torus (the inverse aspect ratio). The shape of the tilted coils depends on both ε and angle β. At $\varepsilon << 1$ the shape of the coils is ellipse with semi axes a and $a/\cos\beta$.



a) b)

Fig. 2. Circular-section torus: a) torus composed of tilted coils; b) current contour configurations at $\varepsilon << 1$.

In order to model a system with tilted coils, the two components of surface current density $\boldsymbol{j}$ must be taken into account. Equations for poloidal j_p and toroidal j_φ components can be easily found from equations (1), (2) and the condition $\operatorname{div} \boldsymbol{j} = 0$. Below the thin toroidal systems is considered. In this case the components are written as

$$\boldsymbol{j} = \boldsymbol{j}_p + \boldsymbol{j}_\varphi = \frac{IN}{2\pi R}\left(\boldsymbol{e}_\theta - \tan\beta\cos\theta\boldsymbol{e}_\varphi\right), \qquad (3)$$

where N is total number of coils in the torus and I is the current per coil (NI is the total ampere turns in the current sheet), $\boldsymbol{e}_\varphi$ and $\boldsymbol{e}_\theta$ are .unit vectors directed along coordinates r and θ.

III. Magnetic fields, energy and forces

In contrast to non-tilted coils ($\beta = 0$), the magnetic field of a tilted coil torus has both toroidal B_φ and poloidal B_p components.

The component B_φ is created by currents j_p and is confined within the torus. The toroidal component has a maximum value at $\rho = R_1 = R - a$ and decreases with increasing distance, ρ, as $B_\varphi = \frac{\mu_0 NI}{2\pi\rho}$.

The component B_p is created by currents j_φ and, in contrast to B_φ, is non-zero both inside and outside of the torus. Inside of torus the magnetic field is almost uniform and it has only vertical component. Outside of thin torus the magnetic fields is determined as magnetic fields created by magnetic moments that distributed along circle line of radius R [6]. A component of vector potential A_φ is given by

$$A_\varphi = \frac{\mu_0 INa^2}{8\pi}\frac{k}{\rho\sqrt{R\rho}}\left[\frac{R^2 - \rho^2 + z^2}{(R-\rho)^2 + z^2}E(k) - K(k)\right], \quad (4)$$

where $K(k)$ and $E(k)$ are complete elliptic integrals of the first and second kind, $k^2 = \frac{4R\rho}{(R+\rho)^2 + z^2}$.

For systems with tilted coils the stray magnetic field has nonzero value. Because the total azimuthal current is equal to zero, far from torus the component B_p is smaller compare to value of stray magnetic field of solenoid with the same total ampere turns. So, for toruses with small ε the stray magnetic field falls as $B_p \sim \varepsilon^2 \tan\beta\left(z^2 + \rho^2\right)^{-3/2}$ and, for example, B_z changes along the z-axis according to the following

$$B_z = B_{z0}\varepsilon^2 \tan\beta\left(1 - \frac{3}{2}\frac{R^2}{R^2 + z^2}\right), \qquad (5)$$

where B_{z0} is the stray magnetic field of a short solenoid with the same ampere turns, IN.

The total energy stored in the magnetic field, W, is the sum of the energy stored in the toroidal, W_φ, and poloidal, W_p, components. At $\varepsilon << 1$ these components are easily determined from model for straight cylinder with surface density current. The total energy is found by the equation

$$W = W_\varphi + W_p = \frac{\mu_0 I^2 N^2 R}{4}\varepsilon^2\left(1 + \frac{1}{2}\tan^2\beta\right) \qquad (6)$$

Each incremental length dl of the coil (Fig. 2b) experiences an electromagnetic force $d\boldsymbol{F} = \boldsymbol{f}dl$, which is perpendicular to the current line. The linear force density, $\boldsymbol{f}$, is given by

$$\boldsymbol{f} = \frac{1}{2}\boldsymbol{I} \times \left(\boldsymbol{B}^+ + \boldsymbol{B}^-\right), \qquad (5)$$

where $\boldsymbol{B}^+$ and $\boldsymbol{B}^-$ are magnetic fields inside and outside of the torus near its surface.

The vector $\boldsymbol{f}$ for a toroidal system with tilted coils has two components

$$\boldsymbol{f} = f_n\boldsymbol{n} + f_b\boldsymbol{b}, \qquad (6)$$

where f_n is the component directed along the external normal, $\boldsymbol{n}$, to the current contour (Fig 2b) and f_b is the component oriented along binormal $\boldsymbol{b}$ perpendicularly to flat of the coil (Fig 2a).

For model of thin torus the equations for magnetic fields near current surface $\boldsymbol{B}^+$ and $\boldsymbol{B}^-$ are the following

$$\boldsymbol{B}^+ = \frac{\mu_0 IN}{2\pi R}\left(\frac{1}{2}\tan\beta\sin\theta\boldsymbol{e}_r + \frac{1}{2}\tan\beta\cos\theta\boldsymbol{e}_\theta + \boldsymbol{e}_\varphi\right)$$
$$\boldsymbol{B}^- = \frac{\mu_0 IN}{2\pi R}\left(\frac{1}{2}\tan\beta\sin\theta\boldsymbol{e}_r - \frac{1}{2}\tan\beta\cos\theta\boldsymbol{e}_\theta\right) \qquad (7)$$

Combining (5) and (7) yields

$$f_n = \frac{\mu_0 I^2 N}{4\pi R}\cdot\frac{\sin\beta\tan\beta\left(1 + \tan^2\beta\sin^2\theta\cos^2\theta\right) + \cos\beta}{\sqrt{1 + \tan^2\beta\cos^2\theta}}$$
$$f_b = \frac{\mu_0 I^2 N}{4\pi R}\cdot\frac{\sin\beta\tan\beta\sin\theta\cos^2\theta}{\sqrt{1 + \tan^2\beta\cos^2\theta}} \qquad (8)$$

In toroidal systems in the case $\beta = 0$ only the normal force density component, f_n, is present. If the coils are tilted, however, both components have to be taken into account. These components are shown in Fig. 3 and Fig. 4 for some values of β.

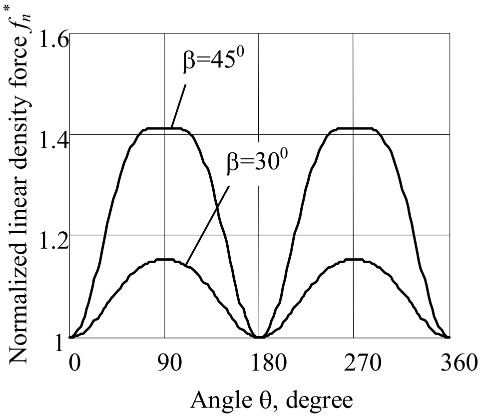


Fig. 3. Distribution of normal linear density force $f_n^* = f_n \dfrac{4\pi R}{\mu_0 I^2 N}$ along perimeter of coil at different tilting angle β.

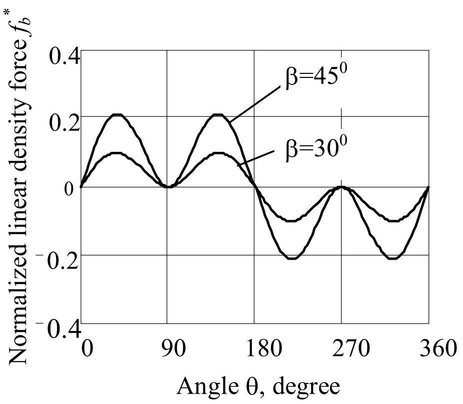


Fig. 4. Distribution of binormal component of density force f_b^* along perimeter of coil at different tilting angle β.

IV. Conclusion

The simple mathematical model of toroidal circular cross-section solenoids with tilted coils was developed. The analytical equations for surface density current, magnetic fields and linear density forces allow to carry out a preliminary analysis of mane features of systems. The analytical model can be used to study the effect of reduction of electromagnetic forces, structural support requirements and other parameters of superconducting magnetic energy storage with tilted coils and another applications of axially symmetric magnetic systems.

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