Mathematical model of action potential propagation in neuron axon

Vsevolod Horyachko, Khrystyna Drohomyretska, Mariya Kotsyuba

***Abstract* — The mathematical model of action potential propagation in neuron axon basing on the electric circuit with distributed parameters and using the straight line method is proposed**

***Index terms* – mathematical model, neuron, action potential**

I. INTRODUCTION

The physiology of nervous system and, in particular, its main structure element – nervous cell for a long time is of great interest for many scientists. The mathematical theory of excitation developed by Nobel prize-winner Hodgkin and Hyxley, basing on the facts of electrophysiological experiments [1] have got universal acknowledgement. Using this theory the model of action potential propagation in nerve cell branch – axon is proposed.

II. THEORETICAL PART

The action potential of axon is rapid and short depolarisation, which is spread in nerve fibre for transmission of electric signal to another nerve fibre, muscular tissue and gland cell.

Owing to their structure and properties nerve fibres are cylindrical conductors. The appearance of action potential in axon is caused by opening and closing of potentialdependant ion canals under the influence of over-threshold stimuli. The propagation of action potential in axon, as electrophysical process, can be described by electric values – voltages and currents, which change along the axon. That why nerve fibre can be presented as electric circuit with distributed parameters as shown at Fig. 1. In such circuit voltages and currents are functions of two variables: time t and distance x.

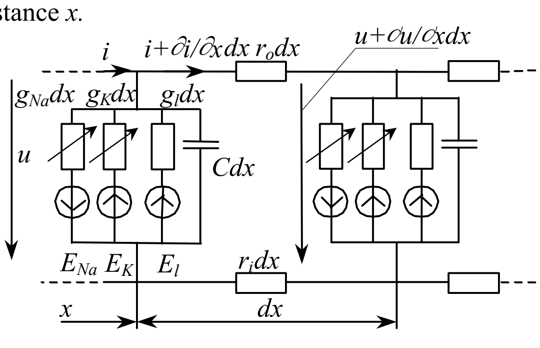

Fig. 1. Electric scheme of axon with distributed parameters

V. Horyachko and M. Kotsyuba are with the Institute of Power Engineering and Control Systems Lviv Polytechnic National University, St. Bandera str., 12, Lviv, 79013, Ukraine

Kh. Drohomyretska is with the Institute of Applied Mathematics and Fundamental Sciences Lviv Polytechnic National University, St. Bandera str., 12, Lviv, 79013, Ukraine

One of them contains electric capacity, the others show natrium, potassium conductance of membrane and the conductance of output. Three electromotive forces are put into last three branches. Let E_l equates the resting potential and the values E_{Na}, E_K are calculated using the Nernst equation

$$E = \frac{RT}{F} \ln \frac{[C^+]_o}{[C^+]_i}, \qquad (1)$$

where E – equilibrium potential; $[C^+]_o$, $[C^+]_{in}$ – ion concentration outside and inside the cell respectively; R – gas constant; T – absolute temperature; F – Faraday number.

As greater and greater depolarizing currents are given, there comes a point at which an entirely different response occurs. Beyond a certain threshold depolarization, the membrane potential suddenly and rapidly decreases to 0 mV and overshoots zero to +35 mV or more. Then, typically within 1-2 ms V_m returns to the resting potential. This brief but dramatic spike of electric activity is called an action potential or nerve impulse.

Parameters g_K, g_{Na} are in complicated dependence on membrane potential and time (Fig. 2) [3]. According to Hodqkin and Huxley conductivity changing is described by non-linear first order differential equation. The approximation of this non-linearities by cubic splines when membrane potential reaches over thresh old level is proposed.

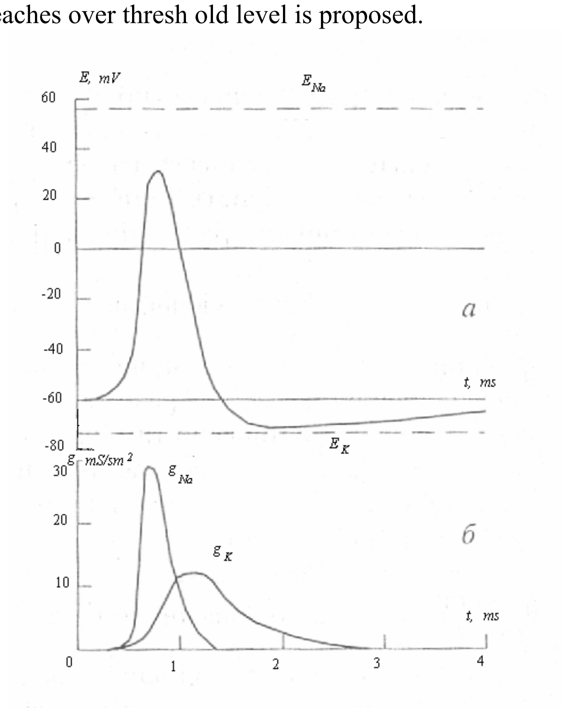

Fig. 2. The dependence of conductance g_K, g_{Na} from membrane potential

Mathematical model of electric circuit with distributed parameters can be described by well-known telegraphic equations. In our situation they are

$$-\partial u / \partial x = (r_o + r_i)i;$$
$$-\partial i / \partial x = g_{Na}(u - E_{Na}) + g_K(u + E_K) + \qquad (2)$$
$$+ g_l(u + E_l) + C\partial u / \partial t,$$

where u – voltage between outside and inside axon parts (membrane potential); i – current along axon; r_o, r_o , g_{Na}, g_K, g_l, C – parameters of electric circuits per unit length.

The parameters of the electric circuits can be got basing on the geometric parameters (internal and external diameter, axon length), physical characteristics of the surroundings and experimentally obtained dependencies.

The equations from system (2) are differential equations in partial derivatives, non-linear and parametrical (taking into account specific of ion conductance g_K, g_{Na}). The straight line method was used for numerical decision of such equations.

The straight line method is method of finite differences for one argument. Such one argument derivative substitution transforms differential equations in partial derivatives with two arguments into ordinary differential equations. Solving the telegraphic equations we apply this method to x argument, that is much more precise than to t argument because the relative coordinate change for (u, i) regime along the nerve fibre is smaller than for time.

Thus using the straight line method the equations for k-interval can be written

$$-(u_{k+1}-u_k)/\Delta x=(r_o+r_i)i_k;$$
$$-(i_k-i_{k-1})/\Delta x=g_{Na}(u_k-E_{Na})+ \qquad (3)$$
$$+g_K(u_k+E_K)+g_l(u_k+E_l)+Cdu_k/dt,$$

where u_{k+1}, u_k – voltage of the beginning and in the end of k-interval; i_k, i_{k-1} – currents of k- and k-1-intervals; Δ – linear integrating step.

The backward differentiation formula (BDF) can be applied for integrating equations (3); the derivative is approximated by discrete analogue [4]

$$(du/dt)_{m+1}=a_0h^{-1}u_{k+1}+h^{-1}\sum_{s=1}^{p}a_su_{k+1-s}\ , \qquad (4)$$

where a_0, a_s – method coefficients; m – number of the time integrating step; h – width of the time integrating step; p – BDF method order.

Using (3) and (4) the single-measured discrete model of action potential propagation in neuron axon was obtained

$$u_{k+1,m+1}=u_{k,m+1}-(r_{\Delta o}+r_{\Delta i})i_{k,m+1};$$
$$i_{k,m+1}=i_{k-1,m+1}-g_{\Delta Na,m+1}(u_{k,m+1}-E_{Na})-$$
$$-g_{\Delta K,m+1}(u_{k,m+1}+E_K)-g_{\Delta l}(u_{k,m+1}+E_l)- \qquad (5)$$
$$-C_\Delta a_0h^{-1}u_{k,m+1}-C_\Delta\sum_{s=1}^{p}a_sh^{-1}u_{k,m+1-s}$$

where

$$r_{\Delta o}=r_o\Delta x;\quad r_{\Delta i}=r_i\Delta x;\quad g_{\Delta Na}=g_{Na}\Delta x;$$
$$g_{\Delta K}=g_K\Delta x;\quad C_\Delta=C\Delta x$$

III. CONCLUSIONS

The worked up mathematical model of action potential propagation in neuron axon can simulate electrophysiological processes in excited nerve cell. Despite the fact that mathematical model basing on the electric circuit with distributed parameters cannot reflect some physical and chemical properties of nervous excitation, with its help one can obtain voltage distributions (membrane potentials) for different time and ion currents along all axon length. In future after working up mathematical model of synaptical connection the modelling of elementary neuron net is planned.

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