Application of invariant approximations of functions to 3-D magnetic field analysis

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***Abstract* — A general technique for analysis of a stationary magnetic field through the application of three-dimensional invariant algebraic analogies of rotor-operator is being presented.**

***Index terms* – magnetic field, three-dimensional invariant, rotor-operator**

I. INTRODUCTION

An apparatus of invariant approximations of functions has been presented in [1]. The apparatus has been used for computation of two-dimensional electro-magnetic fields [2], [3]. However, the issue of applying the apparatus to analysis of 3-D problems stays actual. The presented paper deals with simulation of a 3-D stationary magnetic field based on implementation of the apparatus of invariant approximations of functions.

II. BASIC CONCEPTS

Let us consider a domain G, confined by a boundary Γ. A mathematical model of a boundary problem of stationary magnetic field analysis may be introduced by the following system of differential equations

$$\overline{B}[\overline{r}] = rot\overline{A}[\overline{r}]; \quad (\overline{r} \in G \cup \Gamma) \tag{1}$$

$$rot\overline{H}[\overline{r}] = \overline{J}[\overline{r}]; \quad (\overline{r} \in G) \tag{2}$$

$$\overline{B}[\overline{r}] = \overline{B}[\overline{H}[\overline{r}], \overline{r}]; \quad (\overline{r} \in G \cup \Gamma) \tag{3}$$

$$\overline{A}[\overline{r}] = \overline{A}_{\Gamma}[\overline{r}], \quad (\overline{r} \in \Gamma) \tag{4}$$

where $\overline{B}[\overline{r}]$, $\overline{H}[\overline{r}]$, $\overline{A}[\overline{r}]$ are induction of magnetic field, magnetic tension, and vector magnetic potential, respectively, as sought vector functions of radius-vector $\overline{r}$ of a point located inside G or on its boundary Γ; $\overline{B}[\overline{H}]$ describes a main curve of magnetization of a substance filling the domain G; to emphasize the fact that different areas of the domain may have different magnetic properties the second independent variable $\overline{r}$ was included into the right part of (3); $\overline{A}_{\Gamma}[\overline{r}]$ is vector magnetic potential as a given vector function of radius-vector $\overline{r}$ of a point located on boundary Γ; $\overline{J}[\overline{r}]$ is electrical current density as a given vector function of radius-vector $\overline{r}$ of a point located inside G.

Let us place a grid within the domain G and extend it outside the domain. A principle of the grid configuration is following. Let us begin with nodes located on x-axis with an equal interval h_x between adjacent ones and forming a straight line with space coordinates $y = 0, z = 0$. Therefore, 3-D coordinates of the first line's nodes are $(0,0,0)$, $(h_x,0,0)$, $(2h_x,0,0)$, … ,$(-h_x,0,0)$,$(-2h_x,0,0)$, … The second nodal line with space coordinates $y = h_y, z = 0$ and the third nodal line with space coordinates $y = 0$, $z = h_z$ go in parallel with the first one, but are moved in x-direction for $h_x / 2$. Therefore, x-coordinates of those lines' nodes are fractions of h_x. In general, every new nodal line that is shifted from the old one for one step in y- or z-direction is also displaced in x-direction for $h_x / 2$ in comparison with the old one. After the grid is completed, let us build a minimal encompassing polyhedron in accordance with the rule set forth in [2]. For every external node a corresponding boundary node is being chosen. If a distance between two nodes is less than $0.1h_{min}$, one of them is excluded to avoid singularity of Taylor's matrix. In summary, we have M nodes, among them K internal nodes and Z boundary nodes.

Let us tie to every m-th ($m = \overline{1,M}$) node a sole P-nodal set of the n-th order (where $P = (n+3)!/(n!3!)$) as shown in [1]. We assume that the grid's step is sufficiently little to represent sought variables within a set with Taylor's polynomial of the n-th power. For every set a double numeration has being introduced where the first part of it constitutes the number of a basic node in a nodal numbering and the second part constitutes a local number of a node inside the set. Accordingly to the apparatus of invariant approximations of functions [1] an algebraic analogy of the concerned boundary problem can be written down as

$$\overline{B}_m = \overline{\mathbf{R}}_{\nabla m} \times \overline{\mathbf{A}}_m; \quad (m=\overline{1,M}) \tag{5}$$

$$\overline{\mathbf{R}}_{\nabla m} \times \overline{\mathbf{H}}_m = \overline{J}_m; \quad (m=\overline{1,K}) \tag{6}$$

$$\overline{B}_m = \overline{B}_m[\overline{H}_m]; \quad (m = \overline{1,M}) \tag{7}$$

$$\overline{\mathbf{A}} = C_G \overline{\mathbf{A}}_{\mathbf{G}} + C_{\Gamma} \overline{\mathbf{A}}_{\Gamma}, \tag{8}$$

where $\overline{\mathbf{R}}_{\nabla m}$ is an algebraic analogy of Hamilton's operator for the basic node of the m-th set; $\overline{\mathbf{A}}_m = (\overline{A}_{m1}, \ldots \overline{A}_{mP})_t$, $\overline{\mathbf{H}}_m = (\overline{H}_{m1}, \ldots, \overline{H}_{mP})_t$ are nodal columns of vector magnetic potential and magnetic tension for the m-th set, respectively; $\overline{\mathbf{A}} = (\overline{A}_1, \ldots, \overline{A}_M)_t$; $\overline{\mathbf{A}}_G = (\overline{A}_1, \ldots, \overline{A}_K)_t$; $\overline{\mathbf{A}}_{\Gamma}$

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$=(\overline{A}_{K+1},\ldots,\overline{A}_M)_t$; C_G is a $(K\times M)$-size matrix consisting of two blocks: the first one is a $(K\times K)$-size unit matrix and the second one is a $(K\times Z)$-size matrix with zero elements; C_Γ is $(Z\times M)$-size matrix consisting of two blocks: the first one is a $(Z\times K)$-size matrix with zero elements and the second one is a $(Z\times Z)$-size unit matrix. The system (5)-(8) is nonlinear due to the nonlinearity of the relation (7).

Taking onto account that $B_m=\bar{i}B_{xm}+\bar{j}B_{ym}+$ $+\bar{k}B_{zm}$, $\overline{H}_m=\bar{i}H_{xm}+\bar{j}H_{ym}+\bar{k}H_{zm}$, $\overline{A}_m=\bar{i}A_{xm}+$ $+\bar{j}A_{ym}+\bar{k}A_{zm}$, $\overline{\mathbf{R}}_{\nabla m}=\bar{i}\mathbf{R}_{xm}+\bar{j}\mathbf{R}_{ym}+\bar{k}\mathbf{R}_{zm}$ (where $\mathbf{R}_{xm},\mathbf{R}_{ym},\mathbf{R}_{zm}$ are algebraic analogies of $x-, y-,$ $z-$ derivatives for the basic node of the m-th set), the system (5)-(8) can be rewritten in such a way

$$B_x=\mathbf{R}_{ym}\mathbf{A}_{zm}-\mathbf{R}_{zm}\mathbf{A}_{ym}; \qquad (m=\overline{1,M})$$

$$B_y=\mathbf{R}_{zm}\mathbf{A}_{xm}-\mathbf{R}_{xm}\mathbf{A}_{zm}; \qquad (m=\overline{1,M})$$

$$B_z=\mathbf{R}_{xm}\mathbf{A}_{ym}-\mathbf{R}_{ym}\mathbf{A}_{xm}; \qquad (m=\overline{1,M})$$

$$\mathbf{R}_{ym}\mathbf{H}_{zm}-\mathbf{R}_{zm}\mathbf{H}_{ym}=J_{xm}; \qquad (m=\overline{1,K})$$

$$\mathbf{R}_{zm}\mathbf{H}_{xm}-\mathbf{R}_{xm}\mathbf{H}_{zm}=J_{ym}; \qquad (m=\overline{1,K})$$

$$\mathbf{R}_{xm}\mathbf{H}_{ym}-\mathbf{R}_{ym}\mathbf{H}_{xm}=J_{zm}; \qquad (m=\overline{1,K})$$

$$B_{xm}=B_{xm}\left[\sqrt{H_{xm}^2+H_{ym}^2+H_{zm}^2}\right]; \qquad (m=\overline{1,M})$$

$$B_{ym}=B_{ym}\left[\sqrt{H_{xm}^2+H_{ym}^2+H_{zm}^2}\right]; \qquad (m=\overline{1,M})$$

$$B_{zm}=B_{zm}\left[\sqrt{H_{xm}^2+H_{ym}^2+H_{zm}^2}\right]; \qquad (m=\overline{1,M})$$

$$\mathbf{A}_x=C_G\mathbf{A}_{xG}+C_\Gamma\mathbf{A}_{x\Gamma};$$

$$\mathbf{A}_y=C_G\mathbf{A}_{yG}+C_\Gamma\mathbf{A}_{y\Gamma};$$

$$\mathbf{A}_z=C_G\mathbf{A}_{zG}+C_\Gamma\mathbf{A}_{z\Gamma}. \qquad (9)$$

For getting an algebraic analogy of Hamilton’s operator, sets of the 4th order have been used. In that case we have $P=35$ nodes in a set. Within the set, vector potential may be approximated by the expression

$$\overline{A}=\bar{a}_1+\bar{a}_2x+\bar{a}_3y+\bar{a}_4z+\bar{a}_5x^2/2!+\ldots$$
$$+\bar{a}_{34}yz^3/3!+\bar{a}_{35}z^4/4! \qquad (9)$$

In that case we shall obtain approximation error of the 4th order for analogies of the first grade differential operators (grad, div, rot) and approximation error of the 3rd order for analogies of the second grade differential operators (i.e. Δ). The rule of forming sets for every basic node repeats the rule of forming a Taylor’s row [2]. Sets with exclusively internal nodes were marked as “strictly internal”; sets with a basic node that belonged to the boundary were marked as “boundary”; the others were marked as “adjacent” [3]. For computation of algebraic analogies a local coordinate system (that is a system whose zero point coincides with a basic node) was introduced. The nodal coordinates of a typical strictly internal set are being presented in Table 1.

TABLE 1.

COORDINATES OF A STRICTLY INTERNAL SET IN THE LOCAL COORDINATE SYSTEM FOR $h_x=h_y=h_z=h$

#	1	2	3	4	5	6	7	8	9
x	0	h	0.5 h	0.5 h	-h	1.5 h	1.5 h	0.5 h	0
y	0	0	h	0	0	h	0	-h	h
z	0	0	0	h	0	0	h	0	h

10	11	12	13	14	15	16
0.5h	2h	-0.5h	-0.5h	1.5h	H	1.5h
0	0	h	0	-h	h	0
-h	0	0	h	0	h	-h
17	18	19	20	21	22	23
0	0	0	0	-2h	-1.5h	-1.5h
2h	-h	h	0	0	h	0
h	h	-h	2h	0	0	h
24	25	26	27	28	29	30
-0.5h	-h	-0.5h	h	h	h	h
-h	h	0	0	-h	H	0
0	h	-h	2h	h	-h	2h
31	32	33	34	35		
0	0.5h	0	0.5h	0		
-2h	2h	-h	h	0		
0	h	-h	2h	-2h		

Since nodes do not belong to a surface of the 4-th order, Taylor’s matrix [2] for the said set is nonsingular. Having computed the inverse Taylor’s matrix, we can use (as shown in [1]) following expressions for algebraic analogies of $x-, y-, z-$ derivatives

$$\mathbf{R}_x=\mathbf{T}_2;\ \mathbf{R}_y=\mathbf{T}_3;\ \mathbf{R}_z=\mathbf{T}_4,$$

where $\mathbf{T}_2,\mathbf{T}_3,\mathbf{T}_4$ are the second, third and fourth rows of the inverse Taylor’s matrix calculated in the local coordinate system.

The system (9) was being solved by means of Newton’s method. Therefore, elements of Jacobi’s matrix included magnitudes of differential magnetic permeability of a substance filling the domain.

III. CONCLUSION

A technique for analysis of a stationary magnetic field by means of application of three-dimensional algebraic analogies of rotor-operator has been presented.

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