Interval parameter's identification of the linear dynamic system on the basis of interval data

M. Dyvak, P. Stakhiv, I. Calishchuc

Abstract— **The task of interval parameter's identification of linear dynamic system is considered. Possibility of application of calculation procedures of parameter's interval identification of the static system for the decision of this task is shown. The method of parameter's set identification of the linear dynamic system in form of multidimensional ellipsoids is proposed**.

Index terms— **interval analysis; interval parameter's identification; dynamic systems; interval data.**

I. Introduction

FOR researching of electrical circuits the dynamic models usually are constrain. In this case the tasks of parameter's identification of dynamic system are solve. Last time for identification of parameters both static and dynamic objects interval approach is use. This approach do not require of researching of errors in the channel of measuring. Upper and lower bounds of experimental data – interval data is needed only. For parameter's identification of static objects the high efficiency calculation methods on the basis of selection of the saturated interval data blocks of experiment are developed now. Therefore development of method of interval parameter's identification of the dynamic systems on the basis of application and modification of existent methods of parameter's identification of the static systems is actuality.

II. Statement of Task

We will consider a linear dynamic object, which is described by the system of the discrete equations:

$$\vec{x}_{k+1} = G \cdot \vec{x}_k + Q \cdot \vec{u}_k \,,\ k = 1,2,\dots, \tag{1}$$

$$\vec{y}_{k+1} = C \cdot \vec{x}_{k+1} + \vec{e}_{k+1} \,,\ k = 1,2,\dots, \tag{2}$$

where $\vec{x}_k \in R^m$ is the state variables vector of the system in k discrete time; $\vec{u}_k \in R^n$ is the input variables ("controls") vector in k discrete time; $\vec{y}_{k+1} \in R^p$ is the output variables vector in $k+1$ discrete time; *G, Q, C* –matrices with unknown elements, which must be identified; $\vec{e}_{k+1} = (e_{k+1,1},\dots,e_{k+1,i},\dots,e_{k+1,p})^T$ is the random bounded errors vector, which in $k+1$ discrete time are measured.

Lets the known maximal amplitude of these errors Δ

$$|e_{k+1}| \le \Delta, \Delta > 0 \forall k = 1,2,\dots . \tag{3}$$

Not in contempt of generality we are assumed: $C=I$ – singles matrix, then $p = m$ and we will consider the system with the scalar control, that is $\vec{u}_k = (0,\dots u_k)^T$, $Q = \begin{pmatrix} 0 \\ 0\dots q \end{pmatrix}$.

Then systems (1), (2) we can write down in such kind:

$$\begin{cases} x_{1,k+1} = g_{1,1} \cdot x_{1,k} + \dots + g_{1,i} \cdot x_{i,k} + \dots + g_{1,m} \cdot x_{m,k} \\ . \\ . \\ x_{i,k+1} = g_{i,1} \cdot x_{1,k} + \dots + g_{i,i} \cdot x_{i,k} + \dots + g_{i,m} \cdot x_{m,k} \\ . \\ . \\ x_{m,k+1} = g_{m,1} \cdot x_{1,k} + \dots + g_{m,i} \cdot x_{i,k} + \dots + g_{m,m} \cdot x_{m,k} + q \cdot u_k \end{cases} \tag{4}$$

$$\vec{y}_{k+1} = \vec{x}_{k+1} + \vec{e}_{k+1} \,,\ |e_{k+1}| \le \Delta, \Delta > 0 \forall k = 1,2,\dots \tag{5}$$

Estimation of elements (parameters of dynamic system) of matrices *G* and *Q* on the basis of the data of experiment (interval data) is the basic task of interval parameter's identification. These data got as a result of realization of scalar control and measuring state variables with the bounded errors:

$$u_k \to [\vec{x}^-_{k+1}, \vec{x}^+_{k+1}], \tag{6}$$

where $\vec{x}^-_{k+1} = \vec{x}_{k+1} - \vec{i} \cdot \Delta$ and $\vec{x}^+_{k+1} = \vec{x}_{k+1} + \vec{i} \cdot \Delta$, $k = 1,2,\dots$ are lower and upper bounds of the assured interval of state variable vectors; $\vec{i}$ is the vector, all components of which are "1".

III. Method of Interval Parameter's Identification of Dynamic Object on the Basis of Selection of the Saturated Block of Experiment

Using the data of experiment (6) system (4), (5) we will rewrite in such kind

$$\begin{cases} x^-_{1,k+1} \le g_{1,1} \cdot [x^-_{1,k}, x^+_{1,k}] + \dots + g_{1,m} \cdot [x^-_{m,k}, x^+_{m,k}] \le x^+_{1,k+1} \\ . \\ . \\ x^-_{i,k+1} \le g_{i,1} \cdot [x^-_{1,k}, x^+_{1,k}] + \dots + g_{i,m} \cdot [x^-_{m,k}, x^+_{m,k}] \le x^+_{i,k+1} \qquad k = 1,2,\dots \\ . \\ . \\ x^-_{m,k+1} \le g_{m,1} \cdot [x^-_{1,k}, x^+_{1,k}] + \dots + g_{m,m} \cdot [x^-_{m,k}, x^+_{m,k}] + q \cdot u_k \le x^+_{m,k+1} \end{cases} \tag{7}$$

From at each i row of the system (7), for $k \ge m$, we will get k interval equations which allow to conduct

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identification of coefficients of matrix G, that are found in i row.

Let's consider the system $k=m$ of the interval equations, got on the basis of m row from the system (7):

$$\begin{cases} x_{m,1}^- \le g_{m,1}\cdot[x_{1,0}^-, x_{1,0}^+]+\ldots+g_{m,m}\cdot[x_{m,0}^-, x_{m,0}^+]+q\cdot u_0 \le x_{m,1}^+ \\ . \\ . \\ x_{m,k+1}^- \le g_{m,1}\cdot[x_{1,k}^-, x_{1,k}^+]+\ldots+g_{m,m}\cdot[x_{m,k}^-, x_{m,k}^+]+q\cdot u_k \le x_{m,k+1}^+ \\ . \\ . \\ x_{m,m}^- \le g_{m,1}\cdot[x_{1,m}^-, x_{1,m}^+]+\ldots+g_{m,m}\cdot[x_{m,m}^-, x_{m,m}^+]+q\cdot u_m \le x_{m,m}^+ \end{cases} \quad (8)$$

The interval system of linear algebraic equations (8) is similar to the systems built on the basis of interval data at solve of tasks of parameter's identification of static system models by the method of selection of the saturated block of experiment [2]. In the interval analysis the properties of this system are researched. In particular, is known [2]: in space of coefficient's estimations $\vec{g}_m=(g_{m,1},\ldots,g_{m,m})$ the total solution's set of the system (8) is not convex polyhedron (see an example for the case of m=2, resulted on fig.1).

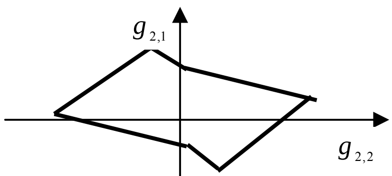

Fig. 1.

The total solution of the interval system (8) is the union of $2^{m\cdot m}$ solutions – the m- dimensional parallelepipeds Ω_{mp} $p=1,\ldots,2^{m\cdot m}$ (see fig. 2. for m=2). Every m- dimensional parallelepipeds Ω_{mp} is the solution's set of the interval system combined from system (8) by using one from $2^{m\cdot m}$ combination of lower or upper bound of intervals $[x_{i,k}^-, x_{i,k}^+], i=1,\ldots m, k=1,\ldots m$.

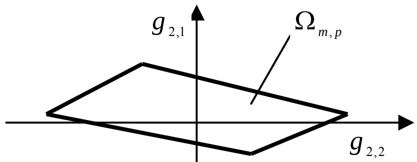

Fig. 2.

Taking into account lemma 1 from work [2], for the got solution as a parallelepiped there is the optimum assured estimation as a m-dimensional ellipsoid:

$$Q_{m,p}=\left\{\vec{g}_m\in R^m \left| (\vec{g}_m-\vec{\bar{g}}_{m,p})^T\cdot X_p^T\cdot E^{-2}\cdot X_p\cdot(\vec{g}_m-\vec{\bar{g}}_{m,p})=m\right.\right\} \quad (9)$$

where X_p is the matrix, built from the lower or upper bounds of intervals $[x_{i,k}^-, x_{i,k}^+], i=1,\ldots m, k=1,\ldots m$, for example, matrix X_p can write down in such

$$X_p=\begin{pmatrix} x_{1,0}^-,\ldots,\ x_{i,0}^+,\ldots,\ x_{m,0}^- \\ . \\ . \\ x_{1,k}^-,\ldots,\ x_{i,k}^-,\ldots,\ x_{m,k}^+ \\ . \\ . \\ x_{1,m}^+,\ldots,\ x_{i,m}^-,\ldots,\ x_{m,m}^+ \end{pmatrix}$$

$\vec{\bar{g}}_{m,p}=X_p^{-1}\cdot\vec{\bar{X}}_{k+1}$ is the center of ellipsoid;

$\vec{\bar{X}}_{k+1}=\left(0.5\cdot(x_{m,1}^-+x_{m,1}^+),\ldots,0.5\cdot(x_{m,m}^-+x_{m,m}^+)\right)^T$;

$E=diag\left\{0.5\cdot(x_{m,1}^+-x_{m,1}^-),\ldots,0.5\cdot(x_{m,m}^+-x_{m,m}^-)\right\}$.

Formal estimation of solution of all system (8) can be written down as a union of m-dimensional ellipsoids (9)

$$Q_m(X_p)=\left\{\vec{g}_m\in R^m \left| (\vec{g}_m-\vec{\bar{g}}_{mp})^T\cdot X_p^T\cdot E^{-2}\cdot X_p\cdot(\vec{g}_m-\vec{\bar{g}}_{m,p})\le m\right.\right\},$$
$$X_p\in[X] \quad (10)$$

where

$$[X]=\begin{pmatrix} [x_{1,0}^-, x_{1,0}^+]\ldots[x_{i,0}^-, x_{i,0}^+]\ldots[x_{m,0}^-, x_{m,0}^+] \\ . \\ . \\ [x_{1,k}^-, x_{1,k}^+]\ldots[x_{i,k}^-, x_{i,k}^+]\ldots[x_{m,k}^-, x_{m,k}^+] \\ . \\ . \\ [x_{1,m}^-, x_{1,m}^+]\ldots[x_{i,m}^-, x_{i,m}^+]\ldots[x_{m,m}^-, x_{m,m}^+] \end{pmatrix}.$$

Fig. 3 illustrates for m=2 estimation of solution's set of the system (8) as a set (10).

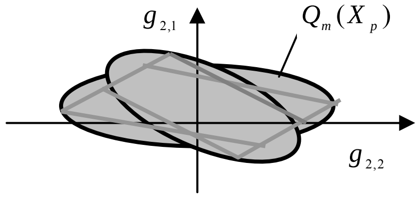

Fig. 3.

Using the got solution (10), the m equation from the system of equations of dynamic system will have such kind:

$$[x_{m,k+1}^-; x_{m,k+k}^+]=[\min_{\vec{g}_m,q\in Q_m(X_p),X_p\in[X]} \vec{g}_m^T\cdot[\vec{x}_k^-,\vec{x}_k^+]+$$
$$+q\cdot u;\ \max_{\vec{g}_m,q\in Q_m(X_p),X_p\in[X]} \vec{g}_m^T\cdot[\vec{x}_k^-,\vec{x}_k^+]+q\cdot u]$$

IV. CONCLUSIONS

1. The analysis of task of interval parameter's identification of the discrete linear dynamic systems showed its similarity of task of parameter's identification of the static systems. Both tasks are erected to finding of solution of the interval system of the linear algebraic equations.

2. The method of interval identification on the basis of selection of the saturated block of experiment is offered. It is allow get the assured parameter's set of dynamic object as an aggregate of ellipsoids. For realization of the offered method algorithms and

calculable procedures of interval parameter's identification of the static systems are suitable.

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