Identification of the internal sources with the aid of boundary element method

Stefan F. Filipowicz

***Abstract* — Electroencephalography (EEG) is a non-invasive method of the brain activity. The principal goal of EEG is to relate measured scalp potentials to current sources generated in brain tissue. In some applications e.g. focal epilepsy, localization of the sources is desired. To solve this problem the Boundary Element Method is used to create the numerical model of the object under consideration. Using synthetic data the inverse (linear in case of EEG) problem will be solved for 2D and 3D space.**

I. INTRODUCTION

The Electroencephalography (EEG) can be used to measure scalp surface potentials. Inverse procedures in EEG are used to estimate the spatial distribution of the underlying, possibly focal, neural sources.

The equivalent current dipoles, and clusters of such dipoles, are a widely used source model for representing focal neural activity. For this model the inverse procedure must estimate the locations and amplitudes of the equivalent dipoles.

EEG problems are similar to the Electric Impedance Tomography (EIT) problems. However there is one exception. In EIT the object under investigation is excited by an external current or voltage source, when in EEG we assume that the exciting source is internal one. Consequence of this is less data which could be collected from the surface. To compare in EIT for 16 electrodes and 8 projection angles we have (16-3)*8=104 linearly independent measurements [8], but in EEG for the same number electrodes we have only 16 measurements. Assuming homogeneity of the region under consideration, the governing equation is a Poisson's equation with the Neumann boundary conditions:

$$\nabla^2 u(\vec{r}) = -b(\vec{r}) \tag{1}$$

$$q(\vec{r}) = \frac{\delta\, u(\vec{r})}{\delta\, n} = 0 \tag{2}$$

where: u – electric potential, b – internal sources, $\vec{r}$ – position vektor

The Inverse Problem in EEG is looking for the localization of the dipoles inside of the region that is why the BEM method is superior over the FEM due to the lack of the internal mesh and troubles with modeling the dipoles. That is why more convenient mathematical model based on integral formulation of Eq.(1) with boundary conditions Eq.(2) was used.

Author is with the Institute of Theory of Electrical Engineering, Measurement and Information Systems, Warsaw University of Technology, ul. Koszykowa 75, 00-662 Warszawa, Poland, e-mail: *2xf@nov.iem.pw.edu.pl*

$$\begin{aligned}\frac{1}{2}u(\vec{r}) + \int_\Gamma \frac{\partial}{\partial n} G(\vec{r},\vec{r}\,')u(\vec{r}\,')d\Gamma = \\ = \int_\Gamma \frac{\partial u(\vec{r}\,')}{\partial n} G(\vec{r},\vec{r}\,')d\Gamma + \int_\Omega bG(\vec{r}_0,\vec{r}\,')d\Omega\end{aligned} \tag{3}$$

In BEM the dipoles are modeled by concentrated sources. They are a special case for which the function b at the internal point $\vec{r}_0$ becomes $q_0 = Q_0\delta(\vec{r}_0)$, where Q_0 is the magnitude of the source and $\delta(\vec{r}_0)$ is a Dirac delta function whose integral is equal to 1 over any volume containing the singularity point r_0 and equal to zero elsewhere. Assuming that inside of the region we have *n* dipoles one can write:

$$\begin{aligned}\frac{1}{2}u(\vec{r}) + \int_\Gamma \frac{\partial}{\partial n} G(\vec{r},\vec{r}\,')u(\vec{r}\,')d\Gamma = \\ = \int_\Gamma \frac{\partial u(\vec{r}\,')}{\partial n} G(\vec{r},\vec{r}\,')d\Gamma + \\ + \sum_{i=1}^{n}\left(q_{0+,i}G(\vec{r}_{0+,i},\vec{r}\,') - q_{0-,i}G(\vec{r}_{0-,i},\vec{r}\,')\right)\end{aligned} \tag{4}$$

where $\vec{r}_{0+}$ and $\vec{r}_{0-}$ are the positions of the charges $+q_0$ and $-q_0$ respectively.

In 2D space Eq.(5a) and for 3D space Eq.(5b), the fundamental solutions of the Eq.(1) are expressed by the Green's functions [2, 3]:

$$G(\vec{r},\vec{r}\,') = \frac{1}{2\pi}\ln\frac{1}{\left|\vec{r}-\vec{r}\,'\right|} \tag{5a}$$

$$G(\vec{r},\vec{r}\,') = \frac{1}{4\pi}\frac{1}{\left|\vec{r}-\vec{r}\,'\right|} \tag{5b}$$

$$\left|\vec{r}-\vec{r}\,'\right| = \sqrt{(x-x')^2 + (y-y')^2} \tag{6}$$

In order to solve the forward problem described by Eq.(4) the BEM was used with zero order boundary elements [2, 3].

The matrix form of Eq.(4) is:

$$Au(\{p\}) = Bq(\{p\}) + Q(\{p\}) \tag{7}$$

Taking into account the boundary conditions (Eq.(2)) Eq.(7) is reduced to:

$$Au(\{p\}) = Q(\{p\}) \tag{8}$$

The matrix A is non-symmetric and fully populated.

II. Objective function definition

EEG Inverse Problem as was mentioned before is looking for the position and the amplitude of the internal sources simulated by dipoles. Those internal sources generate potential on the surface of the object, which in this particular case is the human head. To start with we begin with a simplified 2D model representing the cross section of the real object. So we are looking for such position of the sources for which the potential distribution is as close as possible to the synthetic data we are calling the "measurements". The inverse problem is formulated as an optimization task and the objective function was formulated (see Eq.(9))

$$f_c(\{p\}) = (\{u_c\} - \{u_b\}) * (\{u_c\} - \{u_b\})^T = \\ = \sum_{i=1}^{n} (u_{ci} - u_{bi})^2 \tag{9}$$

$$\{p\} = \{x_1, y_1, x_2, y_2, q\} \tag{10}$$

where: $\{u_c\}$ – electric potentials (on the boundary) depending on the dipole parameters defined by the vector $\{p\}$,

$\{u_b\}$ – measured (or synthetic) potentials on the boundary,

x_1, y_1 – coordinates of the first charge

x_2, y_2 – coordinates of the second charge

q – quantity of the charge

Assuming the arbitrary position of the dipoles (for example the number of dipoles is a prior knowledge about our problem, which help us to select the correct solution from the admissible solution's space) and iteratively we are solving the forward EEG problem checking the value of the objective function and changing the sources position to get potential distribution as close as possible to the "measurements". In order to update the vector $\{p\}$, the BFGS formula was used.

When we are looking for more than one dipole than our problem became ill posed and the prior knowledge (which is problem dependant) is necessary for the correct solutions.

III. Sensitivity analysis

The main task of Sensitivity Analysis is to calculate the gradient of the objective function providing the information about direction of improvement in order to update the unknown vector. Direction of improvement Eq.(11) depends on objective function sensitivity with respect to unknown vector according BFGS formula [1].

$$p_i^{k+1} = p_i^k + \Delta_i^k \tag{11}$$

$$\Delta_i^k = f\left(\frac{\partial f_c^{\,k}}{\partial p_i^k}\right) \tag{12}$$

where: k – number of iterations,

p_i – i-th component of unknown vector

Δ_i – direction of improvement of variable p_i

Differentiating Eq.(8) with respect of i-th component of unknown vector we will get the updating formula expressed by Eq.(14).

$$\frac{\partial f_c^{\,k}}{\partial p_i^k} = \frac{\partial}{\partial p_i^k} \sum_{i=1}^{n} (u_{ci}^k(\{p^k\}) - u_b)^2 = \\ = \sum_{i=1}^{n} 2(u_{ci}^k(\{p^k\}) - u_b) \frac{\partial u_{ci}^k(\{p^k\})}{\partial p_i^k} \tag{13}$$

$$p_i^{k+1} = p_i^k + f\left(\sum_{i=1}^{n} 2(u_{ci}^k - u_b) \frac{\partial u_{ci}^k(\{p^k\})}{\partial p_i^k}\right) \tag{14}$$

There are two ways of calculating gradient of the Objective Function: using numerical approach or using the analytical approach. The first way is simple for implementation but ineffective when number of unknown parameters became large. Analytical approach is more complicated for implementation but is fast and more precise. Precision of the solution is particularly important for EEG.

A. Numerical approach

In order to calculate the direction of improvement we need to calculate the potential sensitivities with respect to each component of the unknown vector (see Eq.(14)). Derivatives are replaced by central differences as it is shown in Eq.(15):

$$\frac{\partial u_{ci}^k(\{p^k\})}{\partial p_i^k} \cong \frac{u_c(\{p^k + \Delta p_i\})}{2\Delta p_i} - \frac{u_c(\{p^k - \Delta p_i\})}{2\Delta p_i} \tag{15}$$

This approach is rather time consuming even in case when vector $\{p^k\}$ is small. To get this results we have to solve the whole problem for unperturbed parameters and next twice for each element of perturbed vector $\{p_i^k\}$. The value of perturbation is problem dependent. In order to get as good results as possible the values of perturbation were determined by numerical experiment. The best results were achieved for the perturbation equal to 0.1% of the value of relevant parameter.

$$\{p_p^k\} = \{x_1^k + \Delta x_1; y_1^k; x_2^k; y_2^k; q^k\} = \{p^k + \Delta p_1\} \tag{16}$$

B. Analytical approach

So called analytical method rely on differentiation of state equation Eq.(8).

$$\frac{\partial}{\partial p_i}(Au(\{p\}) = \frac{\partial Q(\{p\})}{\partial p_i} \tag{17}$$

Due to the fact that matrix A is not dependent on vector parameter's, Eq.(17) could be expressed as follows:

$$A\frac{\partial(u(\{p\})}{\partial p_i} = \frac{\partial Q(\{p\})}{\partial p_i} \quad (18)$$

Solution of Eq.(18) will provide the values needed for direction of improvement calculation. In order to solve Eq.(18) we need to differentiate analytically the right hand side in the following way:

$$Q(\{p\}) \cong \sum_{i=1}^{n} q\frac{1}{2\pi}\ln\frac{1}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \quad (19)$$

$$\frac{\partial Q(\{p\})}{\partial p_1} = \frac{\partial Q(\{p\})}{\partial x_1} =$$
$$= q\frac{1}{2\pi}\frac{\partial}{\partial x_1}\left[\ln\frac{1}{\sqrt{(x_1 - x_j)^2 + (y_1 - y_j)^2}}\right] = \quad (20)$$
$$= q\frac{1}{2\pi}\frac{-(x_1 - x_j)}{(x_1 - x_j)^2 + (y_1 - y_j)^2}$$

This method is much faster and much more precise as we will see in the next section.

IV. Results of numerical experiments

Numerical experiments of localization of dipole modeled by two point sources were carried out for the synthetic data. To avoid so called "inverse crime" the "measurements" were calculated with the aid of BEM but for different discretization than this one used for the Inverse Problem solution.

Two kind of numerical simulation were carried out: for 2D region (circle) and for 3D one (sphere).

In case of 2D space the three dipoles localization problem was solved and results are shown in Fig.1. As we can see in Fig.1, for unpolluted data - "measurements" the localization is perfect.

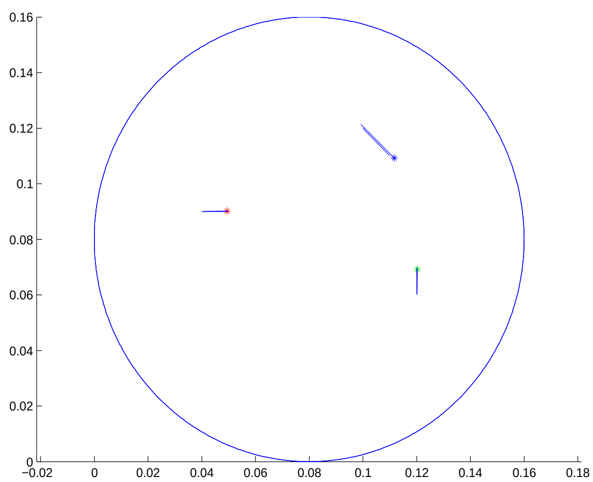

Fig. 1. Three dipoles position reconstruction in case of synthetic pollution free data

The second example for 3D space is localization of one dipole placed inside of the sphere. Using the Boundary Element method [3] as the forward problem solver, results of calculation for the starting position are presented in Fig.2.

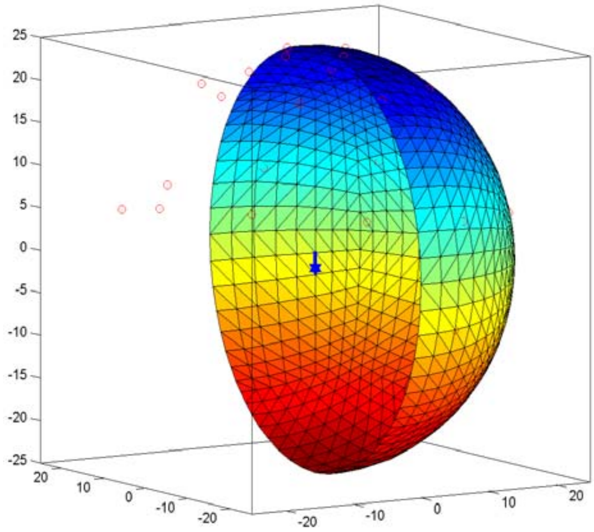

Fig. 2. Potential distribution on the sphere and one dipole starting position inside of the region.

Using this forward solver the Inverse Problem was formulated and preliminary results of the solution are shown in Fig.3. The expecting position of the dipole was (0, 0, 19) for negative charge and (0, 0, 20) for positive charge. All dimensions are in centimeters. The optimization process was terminated on the following positions (5.3, 5.3, 16.1) for negative and (7.3, 7.3, 17.5) for negative charge respectively. So the relative error of one dipole localization is equal to 20%.

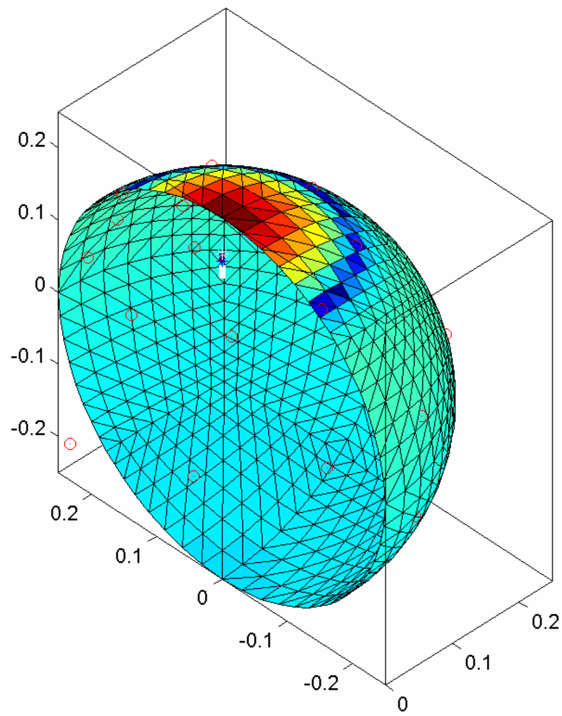

Fig. 3 Expected (black) and achieved (white) position of the dipole inside of the sphere of radius 25cm.

V. FUTURE WORK

The future work will be concentrated on the solution of the inverse problem based on the measured data collected from 2D and 3D phantoms. The bottom hemisphere of the 3D phantom with the semi-rigid coaxial cable as a model of the dipole is shown in Fig.4.

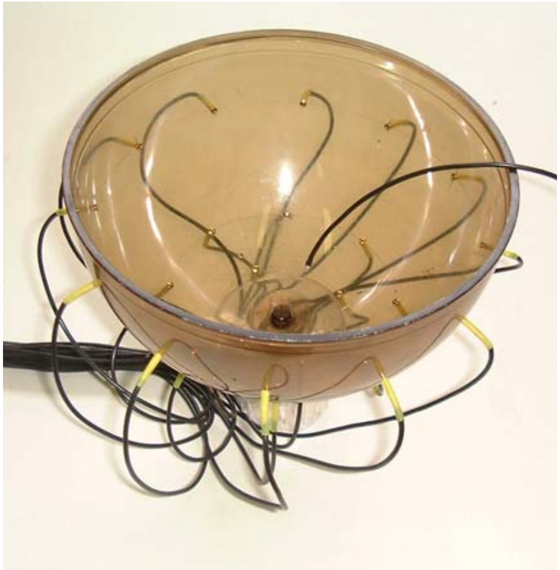

Fig. 4. Example of the 3D phantom with the set of electrodes.

The construction of the dipole is presented in Fig.5.

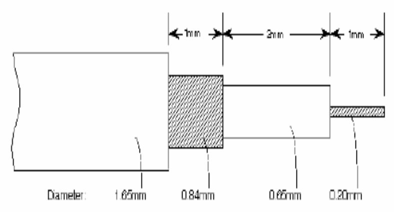

Fig. 5. Dimensions of the stainless steel coax and outer sheath used to generate the dipolar sources.

VI. CONCLUSION

In this paper, the Boundary Element Method applied to the forward problem of localization of the dipoles was shown. The forward problem coupled with the Variable Metric Method allows us to solve efficiently the Inverse Problem. Reported results concern the localization in 2D space of three dipoles, prove that proposed algorithm is efficient and provide the reliable and precise results.

Results of the forward problem for the 3D space are very promising regarding the applications in the Inverse algorithm.

In the future, our work will be concentrated on the Inverse Problem with the measurement collected from the phantom and expansion in 3D space with more realistic model of the human head.

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