Computing Input-Output Characteristics of Circuits Containing Idealized Diodes and Transistors

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***Abstract* — The paper deals with circuits containing bipolar transistors represented by the Ebers-Moll model with idealized diodes. A basic question how to compute efficiently multi-valued input-output characteristics of these circuits is considered. A very efficient algorithm which exploits some ideas of the Katzenelson method is developed and adapted to the idealized case. It is illustrated via two numerical examples of realistic transistor circuits.**

I. Introduction

Finding input-output characteristics is a basic question of the analysis and design of nonlinear electronic circuits [1]-[14]. Hence, the problem arises how to find the characteristics efficiently. There are several methods for tracing the characteristics referring to different classes of circuits [4]-[14]. In this paper transistor circuits with idealized models are analyzed and a very efficient algorithm is developed for computing multi-valued input-output characteristics.

Let us consider a circuit containing the Ebers-Moll modeled bipolar transistors, diodes, linear resistors and independent DC sources. Let z be the input signal (voltage or current source) in the circuit. Under rather mild restrictions the circuit can be described by the Sandberg-Willson equation [15]

$$\boldsymbol{f}(\boldsymbol{v})+\boldsymbol{A}\boldsymbol{v}=\boldsymbol{b} \tag{1}$$

where $\boldsymbol{v}=[v_1 \cdots v_n]^{\mathrm{T}}$ is a vector composed of the emitter-base and collector-base voltages of the transistors and voltages across the diodes; $\boldsymbol{A}=[a_{ij}]$ is an $n\times n$ matrix; $\boldsymbol{b}$ is a source vector which can be presented in the form

$$\boldsymbol{b}=\hat{\boldsymbol{b}}+\boldsymbol{k}z \tag{2}$$

where $\hat{\boldsymbol{b}}$ and $\boldsymbol{k}$ are n-dimension vectors; $\boldsymbol{f}(\boldsymbol{v})=[y_1 \dots y_n]^{\mathrm{T}}=[f_1(v_1)\cdots f_n(v_n)]^{\mathrm{T}}$ is a vector composed of the currents y_j flowing through the diodes, included in the Ebers-Moll model and the other diodes, which originally are specified by equation

$$y_j = f_j(v_j)=K_j\left(\mathrm{e}^{\lambda v_j}-1\right), \qquad (j=1,\dots,n).$$

In this paper all the diodes will be considered as ideal and represented by the two-segment characteristic as shown in Fig. 1 where v_0 is a constant threshold voltage.

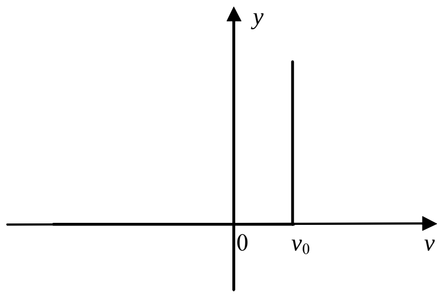

Fig. 1. The characteristic of the ideal diode.

We introduce new variables

$$x_j = v_0 - v_j \geq 0 \qquad j=1,\dots,n \tag{3}$$

and substitute (2) and (3) into (1)

$$\boldsymbol{y}=\boldsymbol{d}+\boldsymbol{k}z+\boldsymbol{A}\boldsymbol{x} \tag{4}$$

where $\boldsymbol{y}=[y_1 \cdots y_n]^{\mathrm{T}}$, $\boldsymbol{x}=[x_1 \cdots x_n]^{\mathrm{T}}$, $\boldsymbol{d}=[d_1 \cdots d_n]^{\mathrm{T}}=\hat{\boldsymbol{b}}-v_0\sum_{j=1}^{n}\boldsymbol{A}_j$, where $\boldsymbol{A}_j$ is j-th column of $\boldsymbol{A}$.

We wish to compute an input-output characteristic, with the input signal z, belonging to a prescribed interval $[z^-, z^+]$.

II. Tracing Input-Output Characteristics

Consider a solution of equation (4) corresponding to a given value of the input z. The solution is represented by two vectors $\boldsymbol{x}=[x_1 \cdots x_n]^{\mathrm{T}}$ and $\boldsymbol{y}=[y_1 \cdots y_n]^{\mathrm{T}}$, which satisfy the relations $x_i \geq 0$, $y_i \geq 0$, $x_i y_i = 0$, and define the diodes states. We say that the i-th diode is in U state if $x_i = 0$ and $y_i \geq 0$ and in strictly U state if $x_i = 0$, $y_i > 0$. Similarly, we say that i-th diode is in L state if $y_i = 0$ and $x_i \geq 0$, and in strictly L state if $y_i = 0$ and

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$x_i > 0$. Note that if $x_i = 0$ and $y_i = 0$ then *i*-th diode is both in *U* state and *L* state.

Assume that the solution corresponding to $z = z_1 = z^-$ is such that each diode is either in strictly *U* state or strictly *L* state. Note that either $x_i = 0$ or $y_i = 0$ for $i = 1, \dots, n$ and substitute these zero values into equation (4).

Let $x_{j_1} = x_{j_2} = \dots = x_{j_p} = 0$ and $y_{l_1} = y_{l_2} = \dots = y_{l_w} = 0$, consequently equation (4) becomes

$$\boldsymbol{y} = \boldsymbol{d} + \boldsymbol{k} z_1 + \begin{bmatrix} \boldsymbol{A}_{l_1} & \boldsymbol{A}_{l_2} & \cdots & \boldsymbol{A}_{l_w} \end{bmatrix} \begin{bmatrix} x_{l_1} \\ x_{l_2} \\ \vdots \\ x_{l_w} \end{bmatrix} \tag{5}$$

where $\boldsymbol{A}_j$ denotes *j*-th column of $\boldsymbol{A}$. Since $y_{l_1} = y_{l_2} = \dots = y_{l_w} = 0$, equation (5) can be split into two matrix equation

$$\begin{bmatrix} y_{j_1} \\ \vdots \\ y_{j_p} \end{bmatrix} = \begin{bmatrix} d_{j_1} \\ \vdots \\ d_{j_p} \end{bmatrix} + \begin{bmatrix} k_{j_1} \\ \vdots \\ k_{j_p} \end{bmatrix} z_1 + \begin{bmatrix} \tilde{\boldsymbol{A}}_{l_1} & \tilde{\boldsymbol{A}}_{l_2} & \cdots & \tilde{\boldsymbol{A}}_{l_w} \end{bmatrix} \begin{bmatrix} x_{l_1} \\ x_{l_2} \\ \vdots \\ x_{l_w} \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} d_{l_1} \\ \vdots \\ d_{l_w} \end{bmatrix} + \begin{bmatrix} k_{l_1} \\ \vdots \\ k_{l_w} \end{bmatrix} z_1 + \begin{bmatrix} \approx{\boldsymbol{A}}_{l_1} & \approx{\boldsymbol{A}}_{l_2} & \cdots & \approx{\boldsymbol{A}}_{l_w} \end{bmatrix} \begin{bmatrix} x_{l_1} \\ x_{l_2} \\ \vdots \\ x_{l_w} \end{bmatrix} \tag{7}$$

where $\tilde{\boldsymbol{A}}_k = \begin{bmatrix} a_{j_1 k} \; a_{j_2 k} \dots a_{j_p k} \end{bmatrix}^{\mathrm{T}}$, $\overset{\approx}{\boldsymbol{A}}_k = \begin{bmatrix} a_{l_1 k} \; a_{l_2 k} \dots a_{l_w k} \end{bmatrix}^{\mathrm{T}}$. The solution to (6) - (7) will be denoted by: $\hat{x}_{l_1}, \dots, \hat{x}_{l_w}$, $\hat{y}_{j_1}, \dots, \hat{y}_{j_p}$.

Now we substitute $z = z_1 + \Delta z$ into equation (4) where $\Delta z > 0$ is sufficiently small, so that all the diodes remain in the same states. Next we split this equation into equations of the form (6) and (7) where z_1 is replaced with $z_1 + \Delta z$ and find their solution. First we solve equation (7) for $x_{l_1}, \dots, x_{l_w}$ and next substitute into (6) to find $y_{j_1}, \dots, y_{j_p}$. If all components of the obtained solution which will be labeled $\hat{\hat{x}}_{l_1}, \dots, \hat{\hat{x}}_{l_w}$, $\hat{\hat{y}}_{j_1}, \dots, \hat{\hat{y}}_{j_p}$ are nonnegative, then all the diodes remain in the same states. Otherwise we discard this solutions and repeat the procedure with $\Delta z < 0$.

Under the assumption that all the diodes are in the same states as for $z = z_1$ the following equations can be written

$$\begin{aligned} x_{l_1} &= \hat{x}_{l_1} + \frac{\hat{\hat{x}}_{l_1} - \hat{x}_{l_1}}{\Delta z}(z - z_1) \\ &\dots\dots\dots\dots\dots \\ x_{l_w} &= \hat{x}_{l_w} + \frac{\hat{\hat{x}}_{l_w} - \hat{x}_{l_w}}{\Delta z}(z - z_1) \\ y_{j_1} &= \hat{y}_{j_1} + \frac{\hat{\hat{y}}_{j_1} - \hat{y}_{j_1}}{\Delta z}(z - z_1) \\ &\dots\dots\dots\dots\dots \\ y_{j_p} &= \hat{y}_{j_p} + \frac{\hat{\hat{y}}_{j_p} - \hat{y}_{j_p}}{\Delta z}(z - z_1) \end{aligned} \tag{8}$$

which are valid as long as all the quantities on the left hand sides are nonnegative.

Let us consider the case where $\Delta z > 0$. We select all the negative coefficients

$$\frac{\hat{\hat{x}}_{l_k} - \hat{x}_{l_k}}{\Delta z}$$

or

$$\frac{\hat{\hat{y}}_{j_k} - \hat{y}_{j_k}}{\Delta z}$$

where $l_k \in \{ l_1, \dots, l_w \}$, $j_k \in \{ j_1, \dots, j_p \}$. We take into account the equations (8) containing the negative coefficients. Next we set to zero the quantities on the left hand sides of these equations and solve each of them for *z*. We choose the minimal *z* and denote it by z_2. Then for $z = z_2$ one of the variables $x_{l_1}, \dots, x_{l_w}$, $y_{j_1}, \dots, y_{j_p}$ becomes equal to zero and all the other are nonnegative. Taking into account the obtained solution corresponding to $z = z_2$ we find a break-point of the characteristic.

Similarly we analyze the case where $\Delta z < 0$.

To find the next break-point we continue the procedure as follows. In the case where x_k was decreased to zero we substitute $x_k = 0$ and introduce y_k as a variable. Otherwise we substitute $y_k = 0$ and introduce x_k as a variable. Next we repeat the described above procedure. The computation process is continued until the break-point corresponding to $z \geq z^+$ is determined. If z^+ is greater than the largest break-point then we introduce $z = z^+$ and find the solution.

Note: If the initial solution contains a pair $x_l = 0$, $y_l = 0$, then we substitute $x_l = 0$ and introduce y_l as a variable. When the computation process is finished we go back to the initial solution and substitute $y_l = 0$ and introduce x_l as a variable. Next the procedure is carried out again. If the initial solution contains several pairs x_j, y_j equal to zero, we consider all the combinations of zero and nonzero values. Similar rule is applied for any other solution obtained during the analysis.

III. NUMERICAL EXAMPLES

To illustrate the algorithm for finding multi-valued input-output characteristics, developed in this paper, we consider below two transistor circuits. All the transistors are represented by the Ebers-Moll model with the idealized diodes and the following parameters: $\alpha_F = 0.99$, $\alpha_R = 0.5$, $V_0 = 0.7\text{V}$, $R_E = R_C = 1\Omega$. The computations are performed on PC Pentium III, 1GHz. The results obtained by the algorithm have been compared with ones provided by SPICE simulator (ICAP/4 Windows, ver. 8.1.7) using the Gummel-Poon transistor model with the parameters IS=6.93fA, BF=99, NF=1, ISE=0, BR=1, NR=1, ISC=0, TNOM=27, and by the algorithm developed in [13].

Example 1

Figure 2 shows the Schmitt trigger containing seven bipolar transistors. We wish to find the transfer characteristic $v_{out} = \hat{v}_{out}(v_{in})$ for $v_{in} = z \in [0, 3]\text{V}$. The input-output characteristic plot obtained by the algorithm developed in this paper is depicted in Fig. 3. The time consumed by the algorithm for tracing the characteristic is less than 1 millisecond. Figure 4 shows the input-output characteristic plot obtained by the algorithm developed in [13]. The characteristic plot provided by SPICE simulator is presented in Fig. 5. The latter one exhibits apparent hysteresis.

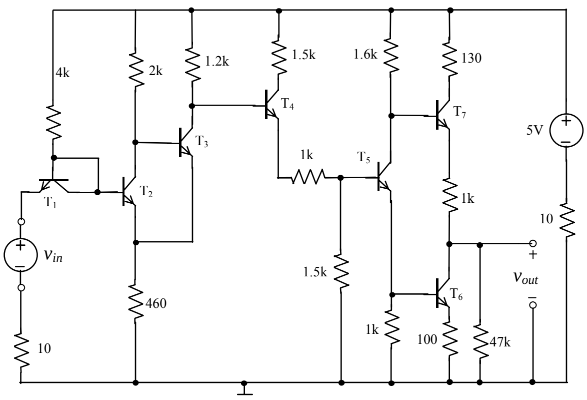

Fig. 2.The Schmitt trigger.

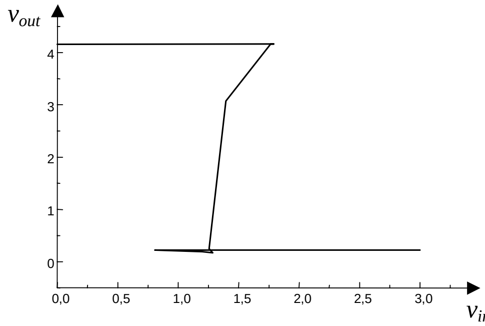

Fig. 3. The input-output characteristic plot obtained by the proposed algorithm.

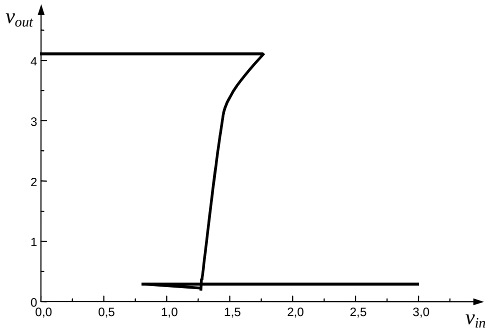

Fig.4. The input-output characteristic plot obtained by the algorithm developed in [13].

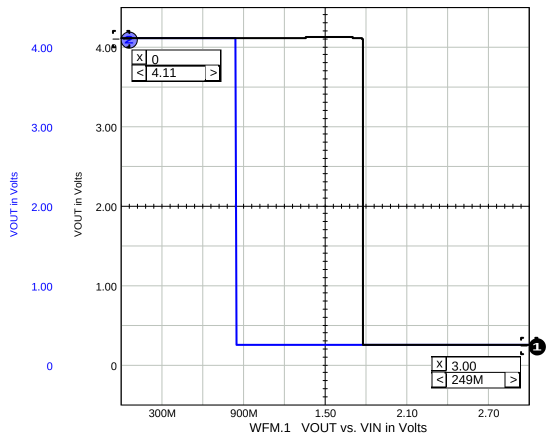

Fig. 5. The characteristic plot provided by SPICE simulator.

Example 2

Figure 6 shows a part of line receiver SN75122 composed of 13 bipolar transistors. We wish to find the transfer characteristic $v_{out} = \hat{v}_{out}(v_{in})$ for $v_{in} = z \in [0, 5]\text{V}$. The input-output characteristic plot obtained by the algorithm developed in this paper is depicted in Fig. 7. The time consumed by the algorithm for tracing the characteristic is less than 1 millisecond. Figure 8 shows the input-output characteristic plot obtained by the algorithm proposed in [13]. The characteristic plot provided by SPICE simulator is presented in Fig.9. Similarly as in Example 1 it exhibits apparent hysteresis.

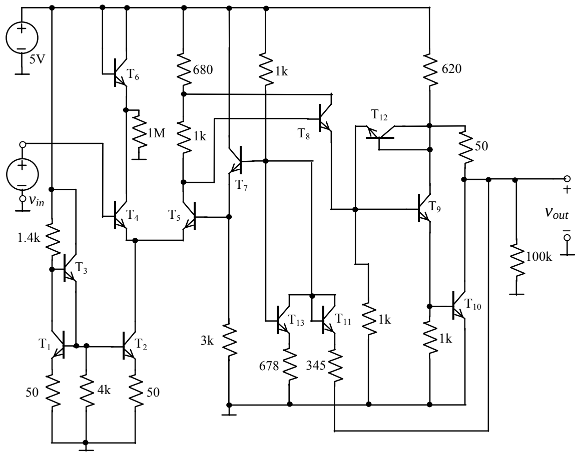

Fig. 6. A part of the line receiver.

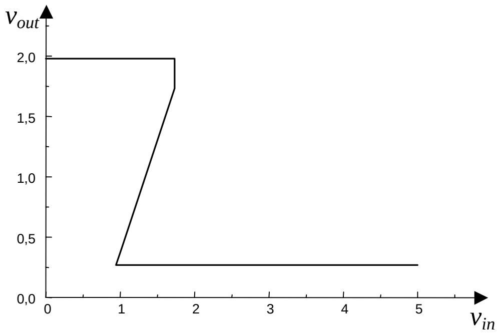

Fig. 7. The input-output characteristic plot obtained by the proposed algorithm.

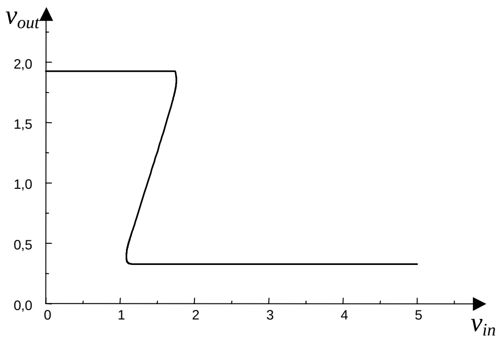

Fig. 8. The input-output characteristic plot obtained by the algorithm developed in [13].

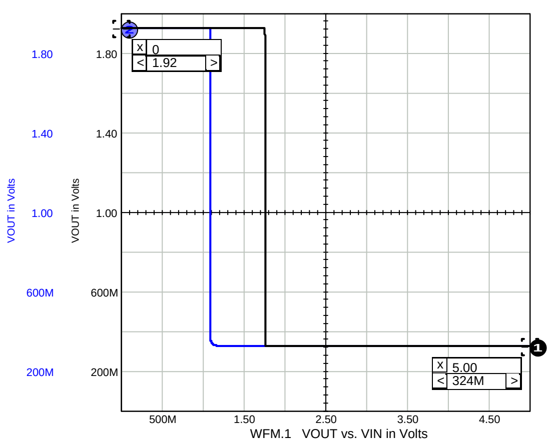

Fig. 9. The characteristic plot provided by SPICE simulator.

IV. Conclusions

The method developed in the paper enables us to trace the multi-valued input-output characteristics in the circuits containing transistors represented by the Ebers-Moll model with idealized diodes. A very efficient algorithm which exploits some ideas of the Katzenelson method [16] is proposed. The obtained characteristic plots are very close to the plots provided by the algorithm developed in [13] which employs the model with exponential type nonlinearities. On the other hand, the SPICE simulator usually provides characteristics exhibiting apparent hysteresis.

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