Higher-Order FDTD (2,6)/(2,8) Method for Calculating Lightning-Induced Voltages on Power Lines

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Abstract **— This paper presents higher-order Finite-Difference Time-Domain (FDTD) methods with 2nd -order of accuracy in time and 6th-, 8th–order of accuracy in space for calculating lightning-induced voltages on power lines. Comparisons between numerical solutions using various numerical schemes are illustrated on graphs. It is shown that the results obtained by using the highest-order scheme have the highest accuracy. This algorithm can be improved as a new basis for solving general EM coupling or telegraphy equations.**

Keywords: **Higher-order FDTD method, overhead power lines, lightning-induced voltage.**

I. Introduction

THE voltages induced by electromagnetic fields radiated by exciting sources as lightning, antenna or a nuclear detonation on power lines represent problems that are considered by many authors all over the world [1]–[4]. The calculation is based on solving EM coupling equations of transmission line using analytical methods [1]–[2] or FDTD method [3]–[4] with the standard scheme (2,2).

In this paper, we proposed a higher-order FDTD method [FDTD(2,6), FDTD(2,8)] for solving EM coupling equations by their discrete approximation at every position along the line and the time. The algorithm is based on higher-order schemes for voltage and current variables expanded by Taylor series after the second order truncating term. The results are discussed.

II. Higher-Order FDTD Schemes

Using Taylor series expansion, we can obtain higher-order finite difference formulae for partial derivatives around grid points of our interest.

The expansions of Taylor series for a function $f(x,t)$ are expressed as:

$$f_{k\pm4} = f_k \pm 4\Delta x f_x + \frac{(4\Delta x)^2}{2!} f_{xx} \pm \frac{(4\Delta x)^3}{3!} f_{xxx} + \frac{(4\Delta x)^4}{4!} f_{xxxx} \pm \frac{(4\Delta x)^5}{5!} f_{xxxxx} + \frac{(4\Delta x)^6}{6!} f_{xxxxxx} \pm \frac{(4\Delta x)^7}{7!} f_{xxxxxxx} + \frac{(4\Delta x)^8}{8!} f_{xxxxxxxx} \ldots, \tag{1}$$

$$f_{k\pm3} = f_k \pm 3\Delta x f_x + \frac{(3\Delta x)^2}{2!} f_{xx} \pm \frac{(3\Delta x)^3}{3!} f_{xxx} + \frac{(3\Delta x)^4}{4!} f_{xxxx} \pm \frac{(3\Delta x)^5}{5!} f_{xxxxx} + \frac{(3\Delta x)^6}{6!} f_{xxxxxx} \pm \frac{(3\Delta x)^7}{7!} f_{xxxxxxx} + \frac{(3\Delta x)^8}{8!} f_{xxxxxxxx} \ldots, \tag{2}$$

$$f_{k\pm2} = f_k \pm 2\Delta x f_x + \frac{(2\Delta x)^2}{2!} f_{xx} \pm \frac{(2\Delta x)^3}{3!} f_{xxx} + \frac{(2\Delta x)^4}{4!} f_{xxxx} \pm \frac{(2\Delta x)^5}{5!} f_{xxxxx} + \frac{(2\Delta x)^6}{6!} f_{xxxxxx} \pm \frac{(2\Delta x)^7}{7!} f_{xxxxxxx} + \frac{(2\Delta x)^8}{8!} f_{xxxxxxxx} \ldots, \tag{3}$$

$$f_{k\pm1} = f_k \pm \Delta x f_x + \frac{(\Delta x)^2}{2!} f_{xx} \pm \frac{(\Delta x)^3}{3!} f_{xxx} + \frac{(\Delta x)^4}{4!} f_{xxxx} \pm \frac{(\Delta x)^5}{5!} f_{xxxxx} + \frac{(\Delta x)^6}{6!} f_{xxxxxx} \pm \frac{(\Delta x)^7}{7!} f_{xxxxxxx} + \frac{(\Delta x)^8}{8!} f_{xxxxxxxx} \ldots \tag{4}$$

$$f_k = f_k . \tag{5}$$

The spatial derivative expressed in the form of a linear combination of the function values at the nodal points are [5]

$$\frac{\partial}{\partial x} f = \alpha_1 . f_{k+2} + \alpha_2 . f_{k+1} + \alpha_3 . f_k + \alpha_4 . f_{k-1} + \alpha_5 . f_{k-2} . \tag{6}$$

From (3)–(6) the fourth-order centered-difference approximations for the first and second-order spatial derivatives of a function $f(x,t)$ at grid point (k,n) are found as [6]

$$\frac{\partial}{\partial x} f(x,t) = \frac{-f_{k+2}^n + 8 f_{k+1}^n - 8 f_{k-1}^n + f_{k-2}^n}{12\Delta x}, \tag{7}$$

$$\frac{\partial^2}{\partial x^2} f(x,t) = \frac{-f_{k+2}^n + 16 f_{k+1}^n - 30 f_k^n + 16 f_{k-1}^n - f_{k-2}^n}{12\Delta x^2} . \tag{8}$$

For finding finite difference formulae of higher accuracy, the spatial derivative can also be expressed in the same way as in (6), with higher-order truncating term given as

$$\frac{\partial}{\partial x} f = \alpha_1 . f_{k+3} + \alpha_2 . f_{k+2} + \alpha_3 . f_{k+1} + \alpha_4 . f_k + \alpha_5 . f_{k-1} + \alpha_6 . f_{k-2} + \alpha_7 . f_{k-3} . \tag{9}$$

From (2)–(5) and (9) we obtain the sixth-order centered-difference approximations for the first and second-order spatial derivatives of a function $f(x,t)$ at grid point (k,n) as

$$\frac{\partial f(x,t)}{\partial x} = \frac{f_{k+3}^n - 9 f_{k+2}^n + 45 f_{k+1}^n - 45 f_{k-1}^n + 9 f_{k-2}^n - f_{k-3}^n}{60\Delta x}, \tag{10}$$

$$\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{180\Delta x^2}\begin{pmatrix} 2 f_{k+3}^n - 27 f_{k+2}^n + 270 f_{k+1}^n - 490 f_k^n \\ +270 f_{k-1}^n - 27 f_{k-2}^n + 2 f_{k-3}^n \end{pmatrix}. \tag{11}$$

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In same way as in (6) and (9), the eighth-order finite difference approximation of the spatial derivative is written as

$$\frac{\partial f}{\partial x}=\alpha_1\cdot f_{k+4}+\alpha_2\cdot f_{k+3}+\alpha_3\cdot f_{k+2}+\alpha_4\cdot f_{k+1} \tag{12}$$
$$+\alpha_5\cdot f_k+\alpha_6\cdot f_{k-1}+\alpha_7\cdot f_{k-2}+\alpha_8\cdot f_{k-3}+\alpha_9\cdot f_{k-4}.$$

From (1)–(5) and (12) we obtain the eighth-order centered-difference approximations for the first and second-order spatial derivatives of a function $f(x,t)$ at grid point (k,n) as [9]

$$\frac{\partial}{\partial x}f(x,t)=\frac{1}{840\Delta x}\begin{pmatrix}-3f_{k+4}^n+32f_{k+3}^n-168f_{k+2}^n+672f_{k+1}^n\\ +3f_{k-4}^n-32f_{k-3}^n+168f_{k-2}^n-672f_{k-1}^n\end{pmatrix} \tag{13}$$

$$\frac{\partial^2}{\partial x^2}f(x,t)=\frac{1}{5040\Delta x^2}\begin{pmatrix}-9f_{k+4}^n+128f_{k+3}^n-1008f_{k+2}^n\\ +8064f_{k+1}^n-14350f_k^n+8064f_{k-1}^n\\ -1008f_{k-2}^n+128f_{k-3}^n-9f_{k-4}^n\end{pmatrix} \tag{14}$$

III. HIGHER-ORDER FDTD FOR EM COUPLING EQUATIONS

A. EM coupling equations of single-phase line

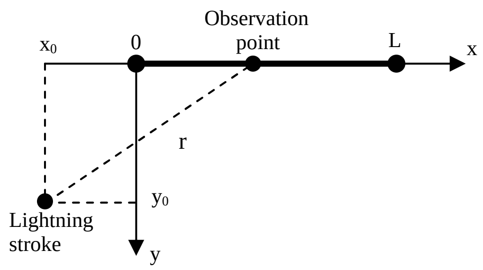

Fig.1. Lightning return stroke nearby transmission line.

The equations of a lossless multiconductor transmission line including EM coupling are expressed as follows [3]:

$$\frac{\partial \mathbf{V}^s(x,t)}{\partial x}+\mathbf{L}\frac{\partial}{\partial t}\boldsymbol{I}(x,t)=\mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t), \tag{15}$$

$$\frac{\partial \mathbf{I}(x,t)}{\partial x}+\mathbf{C}\frac{\partial \mathbf{V}^s(x,t)}{\partial t}=0 \tag{16}$$

where $\mathbf{L}$ and $\mathbf{C}$ are the per-unit length inductance and capacitance matrices of a the three-phase line, see [11], $\mathbf{I}(x,t)$, $\mathbf{V}^s(x,t)$ the column vectors of scattered voltages and currents along the lines and $\mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)$ the column vectors of the incident horizontal electric field along the x axis at the height of the conductor of the three-phase line [2].

After differentiating (15) and (16) with respect to variable x, these EM coupling equations may be rewritten as [3]

$$\frac{\partial^2\mathbf{V}^s(x,t)}{\partial x^2}-\mathbf{LC}\frac{\partial^2\mathbf{V}^s(x,t)}{\partial t^2}=\frac{\partial \mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)}{\partial x}, \tag{17}$$

$$\frac{\partial^2\mathbf{I}(x,t)}{\partial x^2}-\mathbf{CL}\frac{\partial^2\mathbf{I}(x,t)}{\partial t^2}=-\mathbf{C}\frac{\partial \mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)}{\partial t}. \tag{18}$$

Writing the Taylor series for the functions of voltage and current for the time variable with second-order truncating term:

$$\mathbf{V}^s(x,t)=\mathbf{V}^s(x,t_0)+\Delta t\frac{\partial \mathbf{V}^s(x,t)}{\partial t}+\frac{\Delta t^2}{2!}\frac{\partial^2\mathbf{V}^s(x,t)}{\partial t^2}+O(t^3) \tag{19}$$

$$\mathbf{I}(x,t)=\mathbf{I}(x,t_0)+\Delta t\frac{\partial \mathbf{I}(x,t)}{\partial t}+\frac{\Delta t^2}{2!}\frac{\partial^2\mathbf{I}(x,t)}{\partial t^2}+O(t^3). \tag{20}$$

Substituting temporal derivatives of voltage and current from (15)–(18) into (19)–(20) we obtain

$$\mathbf{V}^s(x,t)=\mathbf{V}^s(x,t_0)-\Delta t\mathbf{C}^{-1}\left(\frac{\partial \mathbf{I}(x,t)}{\partial x}\right)+ \tag{21}$$
$$+\frac{\Delta t^2\left(\mathbf{LC}\right)^{-1}}{2LC}\left(\frac{\partial^2\mathbf{V}^s(x,t)}{\partial x^2}-\frac{\partial \mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)}{\partial x}\right)+O(t^3),$$

$$\mathbf{I}(x,t)=\mathbf{I}(x,t_0)+\Delta t\mathbf{L}^{-1}\left(\mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)-\frac{\partial \mathbf{V}^s(x,t)}{\partial x}\right) \tag{22}$$
$$+\frac{\Delta t^2\left(\mathbf{CL}\right)^{-1}}{2}\left(\frac{\partial^2\mathbf{I}(x,t)}{\partial x^2}+\mathbf{C}\frac{\partial \mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)}{\partial t}\right)+O(t^3).$$

B. Solutions Using FDTD(2,6) Scheme

The second-order formula to the first-order temporal derivative of the incident horizontal electric field reads

$$\frac{\partial}{\partial t}\mathbf{E}_{\mathbf{x}}^{\mathbf{i}}(x,h,t)=\frac{\mathbf{E}_{\mathbf{x},k}^{n+1}-\mathbf{E}_{\mathbf{x},k}^{n-1}}{2\Delta t}. \tag{23}$$

We apply sixth-order formulae (10)–(11) to the first and second-order spatial derivatives of voltage and currents and put them into (21)–(22). Now they are written in the FDTD form as

$$\mathbf{V}_k^{n+1}=\mathbf{V}_k^n-\Delta t\mathbf{C}^{-1}\left(\frac{\mathbf{I}_{k+3}^n-9\mathbf{I}_{k+2}^n+45\mathbf{I}_{k+1}^n-45\mathbf{I}_{k-1}^n+9\mathbf{I}_{k-2}^n-\mathbf{I}_{k-3}^n}{60\Delta x}\right)$$
$$+\frac{\Delta t^2\left(\mathbf{LC}\right)^{-1}}{2}\left(\begin{array}{l}\frac{1}{180\Delta x^2}\begin{pmatrix}2\mathbf{V}_{k+3}^n-27\mathbf{V}_{k+2}^n+270\mathbf{V}_{k+1}^n-490\mathbf{V}_k^n\\ +270\mathbf{V}_{k-1}^n-27\mathbf{V}_{k-2}^n+2\mathbf{V}_{k-3}^n\end{pmatrix}\\ -\frac{1}{60\Delta x}\begin{pmatrix}\mathbf{E}_{\mathbf{x},k+3}^n-9\mathbf{E}_{\mathbf{x},k+2}^n+45\mathbf{E}_{\mathbf{x},k+1}^n-45\mathbf{E}_{\mathbf{x},k-1}^n\\ +9\mathbf{E}_{\mathbf{x},k-2}^n-\mathbf{E}_{\mathbf{x},k-3}^n\end{pmatrix}\end{array}\right) \tag{24}$$

$$\mathbf{I}_k^{n+1}=\mathbf{I}_k^n+\Delta t\mathbf{L}^{-1}\left(\mathbf{E}_{\mathbf{x},k}^n-\frac{1}{60\Delta x}\begin{pmatrix}\mathbf{V}_{k+3}^n-9\mathbf{V}_{k+2}^n+45\mathbf{V}_{k+1}^n\\ -45\mathbf{V}_{k-1}^n+9\mathbf{V}_{k-2}^n-\mathbf{V}_{k-3}^n\end{pmatrix}\right)$$
$$+\frac{\Delta t^2\left(\mathbf{CL}\right)^{-1}}{2}\left(\begin{array}{l}\frac{1}{180\Delta x^2}\begin{pmatrix}2\mathbf{I}_{k+3}^n-27\mathbf{I}_{k+2}^n+270\mathbf{I}_{k+1}^n-490\mathbf{I}_k^n\\ +270\mathbf{I}_{k-1}^n-27\mathbf{I}_{k-2}^n+2\mathbf{I}_{k-3}^n\end{pmatrix}\\ +\mathbf{C}\frac{\mathbf{E}_{\mathbf{x},k}^{n+1}-\mathbf{E}_{\mathbf{x},k}^{n-1}}{2\Delta t}\end{array}\right) \tag{25}$$

C. Solutions Using FDTD(2,8) Scheme

We express the first and second-order spatial derivatives of voltage and currents from eighth-order formulae (13)–(14) and insert them into (21)–(22) so that

$$\mathbf{V}_k^{n+1}=\mathbf{V}_k^n-\frac{\Delta t\mathbf{C}^{-1}}{840\Delta x}\begin{pmatrix}-3\mathbf{I}_{k+4}^n+32\mathbf{I}_{k+3}^n-168\mathbf{I}_{k+2}^n+672\mathbf{I}_{k+1}^n\\ +3\mathbf{I}_{k-4}^n-32\mathbf{I}_{k-3}^n+168\mathbf{I}_{k-2}^n-672\mathbf{I}_{k-1}^n\end{pmatrix}$$
$$+\frac{\Delta t^2\left(\mathbf{LC}\right)^{-1}}{2}\left(\begin{array}{l}\frac{1}{5040\Delta x^2}\begin{pmatrix}-9\mathbf{V}_{k+4}^n+128\mathbf{V}_{k+3}^n-1008\mathbf{V}_{k+2}^n\\ +8064\mathbf{V}_{k+1}^n-14350\mathbf{V}_k^n+8064\mathbf{V}_{k-1}^n\\ -1008\mathbf{V}_{k-2}^n+128\mathbf{V}_{k-3}^n-9\mathbf{V}_{k-4}^n\end{pmatrix}\\ -\frac{1}{840\Delta x}\begin{pmatrix}-3\mathbf{E}_{\mathbf{x},k+4}^n+32\mathbf{E}_{\mathbf{x},k+3}^n-168\mathbf{E}_{\mathbf{x},k+2}^n\\ +672\mathbf{E}_{\mathbf{x},k+1}^n-672\mathbf{E}_{\mathbf{x},k-1}^n+168\mathbf{E}_{\mathbf{x},k-2}^n\\ -32\mathbf{E}_{\mathbf{x},k-3}^n+3\mathbf{E}_{\mathbf{x},k-4}^n\end{pmatrix}\end{array}\right) \tag{26}$$

$$\mathbf{I}_k^{n+1} = \mathbf{I}_k^n + \Delta t \mathbf{L}^{-1} \begin{pmatrix} \mathbf{E}_{\mathbf{x},k}^n \\ -\dfrac{1}{840\Delta x}\begin{pmatrix} -3\mathbf{V}_{k+4}^n + 32\mathbf{V}_{k+3}^n - 168\mathbf{V}_{k+2}^n \\ +672\mathbf{V}_{k+1}^n - 672\mathbf{V}_{k-1}^n \\ +168\mathbf{V}_{k-2}^n - 32\mathbf{V}_{k-3}^n + 3\mathbf{V}_{k-4}^n \end{pmatrix} \end{pmatrix} + \frac{\Delta t^2 (\mathbf{CL})^{-1}}{2} \begin{pmatrix} \dfrac{1}{5040\Delta x^2}\begin{pmatrix} -9\mathbf{I}_{k+4}^n + 128\mathbf{I}_{k+3}^n - 1008\mathbf{I}_{k+2}^n \\ +8064\mathbf{I}_{k+1}^n - 14350\mathbf{I}_k^n + 8064\mathbf{I}_{k-1}^n \\ -1008\mathbf{I}_{k-2}^n + 128\mathbf{I}_{k-3}^n - 9\mathbf{I}_{k-4}^n \end{pmatrix} \\ +\mathbf{C}\dfrac{\mathbf{E}_{\mathbf{x},k}^{n+1} - \mathbf{E}_{\mathbf{x},k}^{n-1}}{2\Delta t} \end{pmatrix} \tag{27}$$

D. Boundary conditions:

The boundary conditions at two line terminations for scattered voltages are

$$\mathbf{V}_{k=0}^n = \int_0^h \mathbf{E}_{\mathbf{z}}^i(0,0,t)dz - \mathbf{Z_0}.\mathbf{I}_{k=0}^n , \tag{28}$$

$$\mathbf{V}_{k=L}^n = \int_0^h \mathbf{E}_{\mathbf{z}}^n(L,0,t)dz + \mathbf{Z_L}.\mathbf{I}_{k=L}^n \tag{29}$$

where $\mathbf{E^i_z}(x,z,t)$ is an incident vertical electric field, see [2] and $\mathbf{Z_0}$, $\mathbf{Z_L}$ denote the impedance matrices at two line terminations.

IV. TOTAL INDUCED VOLTAGES

The total induced voltage at each observation point along the line can be expressed by the sum of scattered voltage and the finite integral of incident vertical electric field that is also called incident voltage, see [1]

$$\mathbf{V_T}(x,t) = \mathbf{V}^s(x,t) - \int_0^h \mathbf{E}_{\mathbf{z}}^i(x,h,t)dz = \mathbf{V}^s(x,t) + \mathbf{V}^i(x,t) \tag{30}$$

where $\mathbf{V^i}(x,t)$ is the incident voltage.

V. NUMERICAL RESULTS

We consider an example [1], in which the height of the overhead line is 10 m and its length is 1 km. The lightning striking point has x_0 = -500 m and y_0 = 50 m as indicated in Fig. 1. This current has a peak value of 12 kA, a maximum time-derivative of 50 kA/µs, a return-stroke velocity of $1.3 \cdot 10^8$ m/s. Components of the horizontal and vertical fields of lightning are calculated from analytical formulations in [2].

Calculated results of three components of lightning-induced voltages at observation points along the lossless transmission line using (2,8) FDTD method are illustrated in Figs. 2–4. Curve 3 is the scattered voltage that is computed from EM coupling equation of transmission line adding boundary conditions at two line terminations, curve 2 is the incident voltage that is computed from the incident vertical electric field and curve 1 is the total induced voltage that is calculated by sum of curves 2 and 3. In these figures we can show that the components of the incident voltage and total induced voltage are always positive waveforms, but the component of scattered voltage can be unipolar or negative waveforms that depend on observation points along the line.

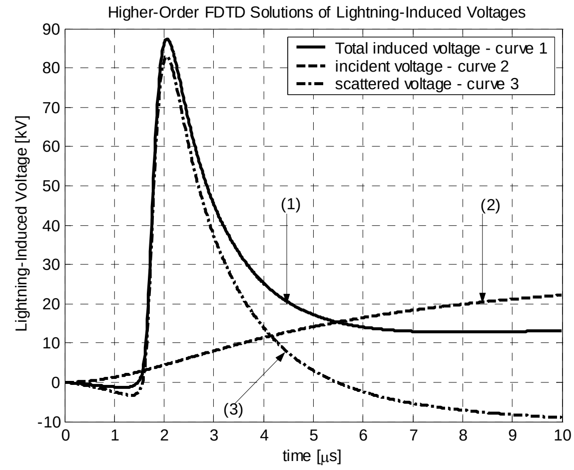

Fig.2. Higher-Order (2,8) FDTD solutions of induced voltages at x = 0 m

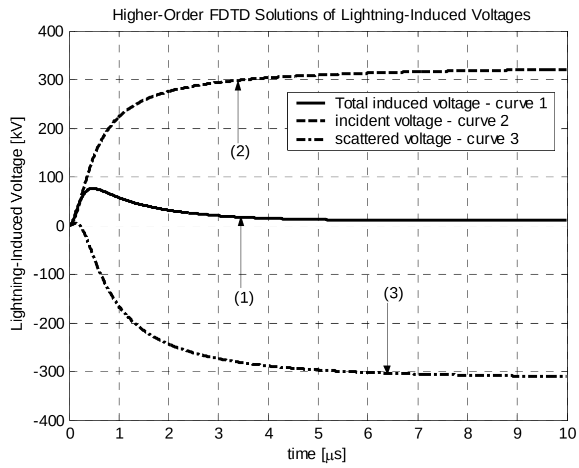

Fig.3. Higher-Order (2,8) FDTD solutions of induced voltages at x = 500 m

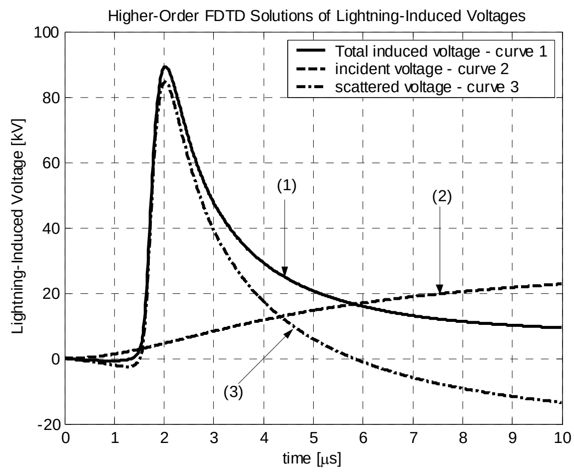

Fig.4. Higher-Order (2,8) FDTD solutions of induced voltages at x = 1000 m

Results of comparisons between various higher-order FDTD solutions of total induced voltage at three observation points along the line are illustrated in following Figs .5–7. Curve 4 is a solution using (2,2) FDTD scheme, curve 3 uses (2,4) FDTD scheme, curve 2 uses (2,6) FDTD scheme and finally curve 1

(2,8) FDTD scheme. It can be seen that the error between solutions of peak values is largest at two line terminations and will decrease toward the middle of the line. (2,8) FDTD scheme for computing lightning induced voltages provides the highest accuracy.

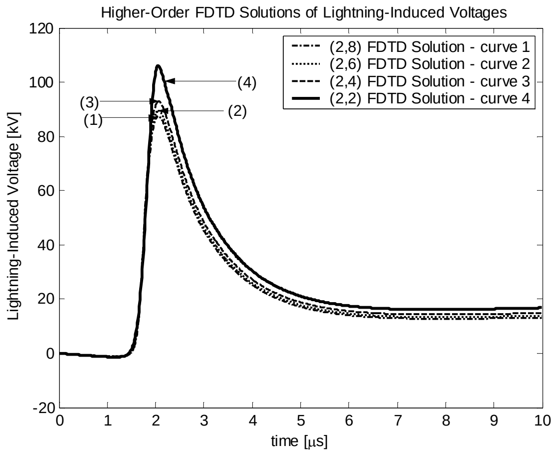

Fig.5. Comparison between (2,2), (2,4), (2,6) and (2,8) FDTD solutions of total induced voltages at x = 0 m

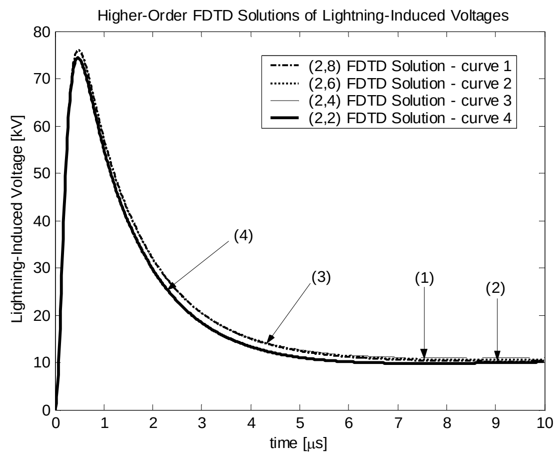

Fig.6. Comparison between (2,2), (2,4), (2,6) and (2,8) FDTD solutions of total induced voltages at x = 500 m

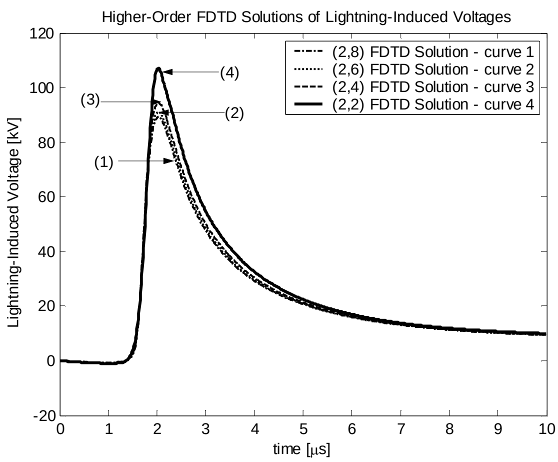

Fig.7. Comparison between (2,2), (2,4), (2,6) and (2,8) FDTD solutions of total induced voltages at x = 1000 m

VI. CONCLUSION

The FDTD methods with standard scheme are popular for computing technical problems. In this paper, higher-order FDTD formulae derived from the Taylor series expansion with second-order accuracy in time and fourth-, sixth-, eighth-order accuracy in space are presented. However, the fourth-, sixth- and eighth-order finite difference formulae of the temporal derivatives are found in the same way as the spatial derivatives. They can be used for calculating problems in fields of mechanical engineering, fluid dynamics, electromagnetic fields… It can be shown that the calculated results reach the highest accuracy when we use higher-order (2,8) FDTD scheme. This algorithm can be used for solving general EM coupling or telegraphy equations on lossy or lossless transmission lines above ground of finite electrical conductivity.

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