The Analysis of the Evolutionary Algorithms for Network Synthesis

Robert Siroić, Stanisław Osowski

***Abstract* – The paper considers the application of the evolutionary algorithms for the synthesis of electrical circuits and networks. The main emphasis will be put on the analysis of the algorithms belonging to the group of evolutionary programming. Different aspects of the implementation of the evolutionary algorithms to the synthesis of the discrete time systems will be presented and discussed.**

I. INTRODUCTION

The evolutionary algorithms belong to the reasonably well developed group of stochastic optimization methods, capable of reaching the global minimum of the optimized objective function [1-7]. Nowadays we can recognize the following main trends of their development.

- Genetic algorithms initiated by J. Holland [1] in the sixties of 20^{th} century as the mathematical counterpart of the natural genetic processes. They employ three main operations: selection, recombination and mutation to build the new generation of the population.
- Evolutionary strategy, direction developed by D. Fogel [5,6] based also on the natural evolution and employing mainly two operations: selection and mutation.
- Genetic programming proposed in 1990 by J. Koza from Berkeley [4]. It is also supported by rules of natural evolution, but directed for the search of the computer program optimally suited for the solved problem.
- Evolutionary programming of Rechenberg, H.. Schwefel and L. Fogel [2,3,7] employing only two operations: selection and mutation (no recombination). In this approach there are specially defined evolutionary parameters forming vector $\boldsymbol{\eta}$, controlling the process of evolution.

In this paper we will study the evolutionary programming and different aspects of its applications to the synthesis of discrete time dynamic systems. The main task in the problem is to find the optimal parameters of the discrete transfer function well adjusted to the given frequency characteristics. The evolutionary algorithm is employed to minimize the objective function, defined as the sum of squares of the differences between the given and actual frequency characteristics at the set of frequency points.

Authors are with the Institute of the Theory of Electrical Engineering, Measurement and Information Systems, Warsaw University of Technology, Warsaw, Poland e-mail: sto@iem.pw.edu.pl

II. EVOLUTIONARY ALGORITHMS

The evolutionary optimization methods started from the genetic algorithm proposed by J. Holland in 1960-ties. Its main task was to maximize the so called fitness function f($\mathbf{x}$), strictly related to the minimized objective function.

The genetic algorithm can be presented in the following form [1]. Assume t=0 as the initial generation. Let us start process with the initial population $\mathbf{x}(t)$, with $\mathbf{x}$ formed by the optimized parameters x_i. The population size (the number M of vectors $\mathbf{x}$ taking part in evolution**)** is constant within the optimization process and is adjusted experimentally. The genetic algorithm is formed by the sequence of the following operations []

- recombination: $\mathbf{x}'(t)= r(\mathbf{x}(t))$
- mutation: $\mathbf{x}''(t)= m(\mathbf{x}'(t))$
- evaluation of the parents and off-springs on the basis of their fitness function $f(\mathbf{x})$
- selection: $\mathbf{x}(t+1)= s(\mathbf{x}''(t))$ leading to the new generation (t+1)

These genetic operations are repeated until the conditions of stopping of the algorithm are met. The quality of final solution strongly depends on many factors, from which the most important is the population size, the recombination and mutation probability, way of coding the optimized variables, selection method applied in forming the new generation and so on. In practice the genetic algorithms employ mostly binary coding of the variables and are not well suited for the optimization of the electrical circuits, where decimal coding of parameter values is more suitable.

In our solution we apply the evolutionary programming strategy of Rechenberg, Schwefel and Fogel [3,5,7] which is relied on the real value (decimal) representation of all variables. It is composed of the following stages.

- Initialization: determination of the population size, generation of the initial values of the optimized vectors $\mathbf{x}$ forming population and evolution vectors $\boldsymbol{\eta}$ corresponding to the vectors $\mathbf{x}$.
- Determination of the fitness functions $f(\mathbf{x}_i)$ for i=1,2, …, M corresponding to each vector $\mathbf{x}$ of current population.
- Generation of the new i^{th} population of the evolution vector $\boldsymbol{\eta}$ and optimized vector $\mathbf{x}$. It is performed for each jth component of both vectors on the basis of the following relations

$$\eta_i^{/}(j) = \eta_i(j) \cdot e^{\tau^{/} \cdot N(0,1) + \tau \cdot N_j(0,1)} \quad (1)$$

$$x_i^{/}(j) = x_i(j) + \eta_i^{/} \cdot N_j(0,1) \quad (2)$$

where:

N(0,1) – random number of normal distribution of zero mean and unity variance, common for ith vector

$N_j(0,1)$ – random number of normal distribution of zero mean and unity variance generated independently for each component j of both vectors: ***x*** and $\boldsymbol{\eta}$.

The coefficients τ and $\tau^{/}$ used in the typical evolutionary algorithm are described by the relations

$$\tau = \frac{1}{\sqrt{2 \cdot \sqrt{n}}} \quad (3)$$

$$\tau^{/} = \frac{1}{\sqrt{2 \cdot n}} \quad (4)$$

where *n* is the dimension of the optimized vector **x** (called also chromosome).

- Determination of the fitness functions for each vector **x**' of the new generated vectors.
- Comparison of the fitness functions of both populations (parents and off-springs) and generation of the new population formed of the best individuals from both populations.
- If the stopping conditions are still not met repeat the procedure of the generation of new population, else assume the best individual as the solution to the optimization problem.

III. THE ANALYSIS OF PERFORMANCE OF THE ALGORITHM

We have applied the presented above evolutionary programming algorithm to the synthesis of the system described by the discrete function H(z). The system should have the frequency characteristics as close as possible to the required magnitude characteristics of the system. The optimized transfer function is assumed of the following form [8,9]

$$H(z) = \frac{b_M z^{-M} + b_{M-1} z^{-(M-1)} + \ldots + b_1 z^{-1} + b_0}{a_N z^{-N} + a_N z^{-(N-1)} + \ldots + a_1 z^{-1} + 1} \quad (5)$$

The optimized parameters are the coefficients a_i if the denominator and b_i of the numerator. The objective function is defined as the sum of squares of the differences of the desired and actual magnitude frequency characteristics *H(f)*, where $H(f) = H(e^{j2\pi f T_s})$ over the assumed frequency range. If the actual response is within the acceptable limit the error is assumed zero. Fig. 1 presents the result of application of the evolutionary algorithm for the design of the 12th order filter of the attenuation of 3dB in the pass band and minimum 40dB in the stop band.

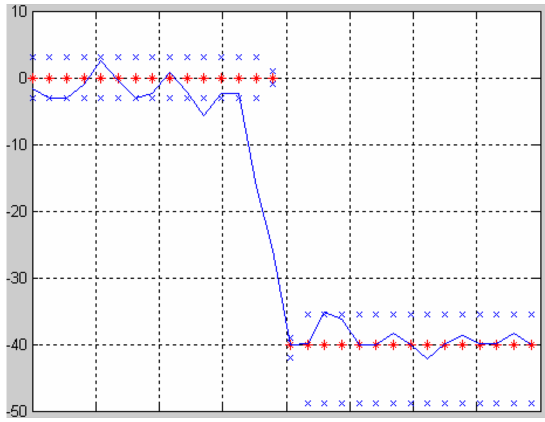


Fig. 1 The desired and actual magnitude characteristics of the filter

The result of application of the algorithm depends very strongly on the initial values of the parameters and the population size. Since we don't know in advance their optimal values, the algorithm may be stuck in the local minimum, far from optimality. To avoid such cases we have applied the reset procedure. In the reset stage we substitute the actual evolutionary vectors $\boldsymbol{\eta}$ by the random values. At the same time we change the chromosomes (the vectors **x**) by multiplying them by the random value close to one. Fig. 2 presents typical change of the objective function (the inverse of the fitness) of the best chromosome resulting from the resetting of the parameters. The resetting in the stagnation phase has enabled to continue the optimization process and finish the procedure in the best possible solution point.

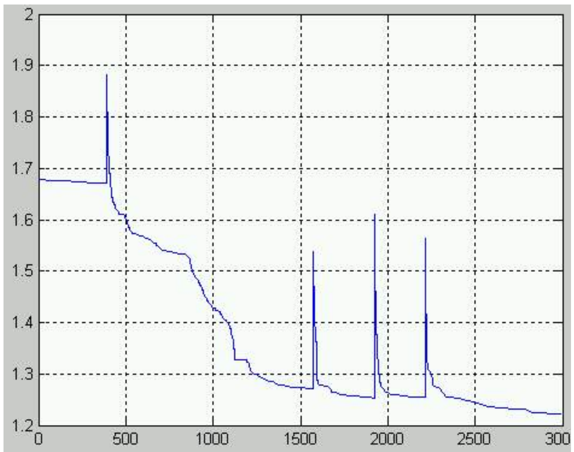


Fig. 2 The change of the objective function in the succeeding generations resulting from resetting

The next problem encountered in the optimization process is the natural tendency of the evolutionary coefficient η to drop to very small values close to zero, ending any change of the chromosomes in the evolution process.

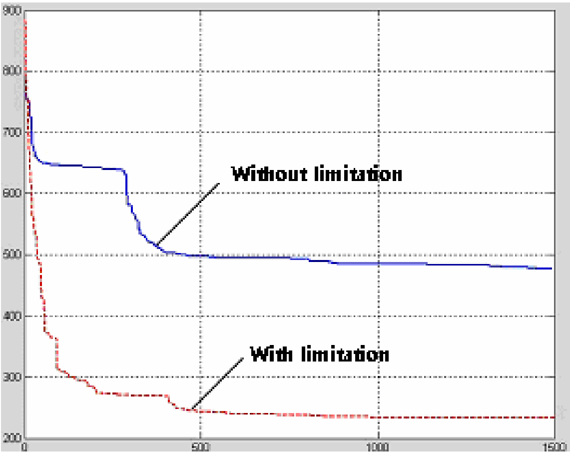


Fig. 3 The comparison of performance of the evolutionary algorithm with and without limitation for the change of η

Close observation of the process has revealed that η can be as small as 10^{-10} in quite early stage of evolution. It means the premature stopping of the algorithm. To avoid this problem w have applied the lower limits of adaptation of η. Except the final stages of the evolution this limit was set on the level of $1e^{-4}$. Only at the end of process the value of η is changing without any limitations. The superiority of the algorithm with assumed limitations of the values of the evolution coefficient is evident. The progress of optimization is much quicker and the minimum of the objective function achieved by the algorithm is deeper.

In the basic implementation of the evolutionary algorithm the normal Gaussian distribution is usually applied. There are another distributions well suited for the task, for example Cauchy. The approach applying both Gaussian and Cauchy distribution is called multi-operator method. The multi-operator algorithm can be presented as following

- In i^{th} generation the following parameters, governing the evolution process are additionally generated for j^{th} optimized variable

$$pq_{ij}^{/} = pq_{ij} + \alpha \cdot N_j(0,1) \qquad (6)$$

$$p_{ij}^{/} = p_{ij} + pq_{ij}^{/} \cdot N(0,1) \qquad (7)$$

where $\alpha = \dfrac{1}{\sqrt{n}}$.

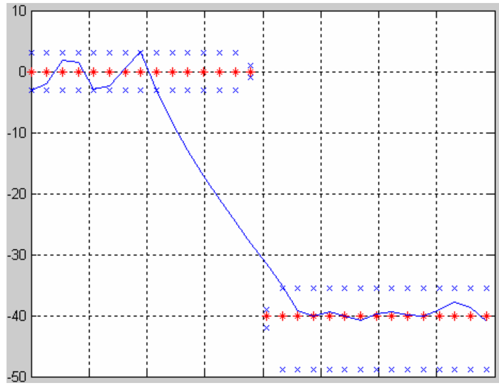
The probability of using the Cauchy distribution in adaptation of optimized j^{th} variable is determined by using the following relation

$$P_C = 1 - \frac{p_{i,j}^{/}}{p_{i,j}^{/} + p_{i,j+1}^{/}} \qquad (8)$$

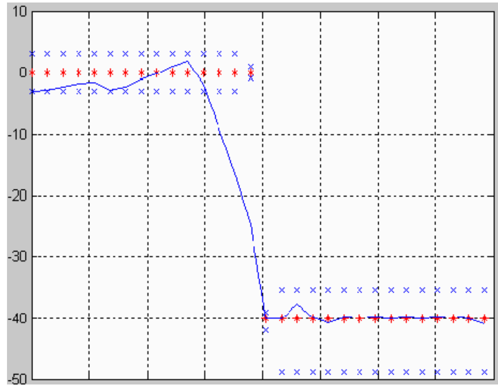
At the same time the evolution coefficient η for jth variable in ith generation is determined using modified form

$$\eta_i^{/}(j) = \beta \cdot \eta_i(j) \qquad (9)$$

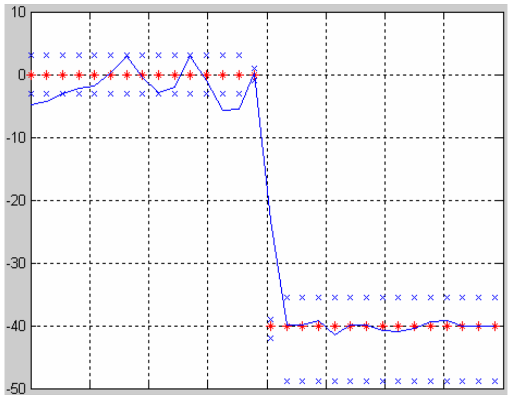
The multi-operator greatly enhances the evolution process. Fig. 4 compares the evolution process at the application of different strategies: normal algorithm with Gaussian distribution (a), normal algorithm with Cauchy distribution (b), and multi-operator application (c) for the same problem of approximation of the given magnitude characteristics by the rational discrete function of 8^{th} order. The superiority of multi-operator is evident. The normal algorithm applying Gaussian distribution leads to the worst results (the widest transient range, not satisfying the requirements). It should be avoided in practical implementation of the algorithm.



a)



b)



c)

Fig. 4 The influence of the distribution of the variables in the evolution process: a) Gaussian, b) Cauchy, c) multi-operator

The next experiment was a bit more demanding. This time we have to synthesize the system of the transfer function of the form (5) to follow the required frequency magnitude characteristics presented in Fig. 5 as close as possible.

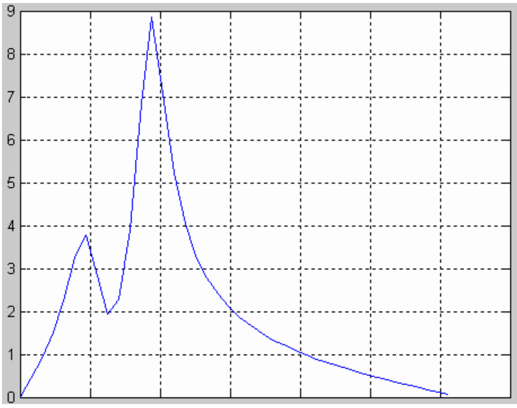


Fig. 5 The required magnitude characteristics in the test

In solving this task we have applied the most efficient implementation of the algorithm with the multi-operator. The degree of the transfer function used in experiment was equal 5 (11 optimized variables). Fig. 6 illustrates the best obtained result of the evolutionary algorithm for this problem.

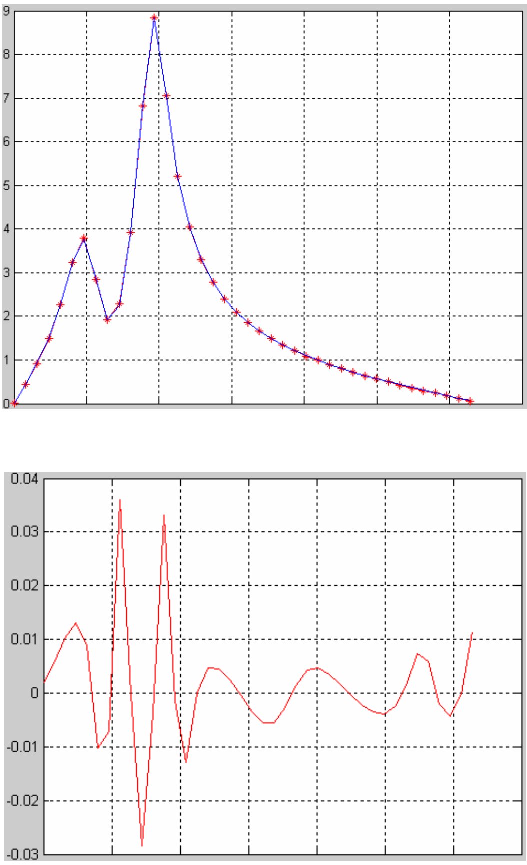


Fig. 6 The results of the test: a) the desired and actual magnitude characteristics, b) the error of realization

The results presented in Fig. 6 confirm that the evolutionary algorithm is capable of solving this difficult synthesis problem with good, acceptable accuracy.

IV. CONCLUSIONS

The paper has been devoted to the analysis of performance of the evolutionary algorithms for the synthesis task of the networks in the frequency domain. It has been shown that standard implementation of the Rechenberg algorithm may be greatly enhanced by the introduction of the so called multi-operator, combining the Gaussian and Cauchy distribution. Close analysis of the results have confirmed that evolutionary algorithms may be in the future good supplement of the well developed family of design methods for discrete filters.

REFERENCES

1. D. E. Goldberg, Algorytmy genetyczne i ich zastosowania, WNT Warszawa 1995
2. Z. Michalewicz, Algorytmy genetyczne i ich zastosowania, WNT Warszawa 1996
3. R. Haupt, S. Haupt, Practical genetic algorithms, Wiley, 1998
4. J. Koza et.al., Genetic Programming IV: Routine Human-Competitive Machine Intelligence, Kluwer Academic Publishers , 2003
5. T. Baeck, D. Fogel, and Z. Michalewicz (editors), Evolutionary Computation: Basic Algorithms and Operators, Institute of Physics, London, 2000
6. T. Baeck, D. Fogel, and Z. Michalewicz (editors), Evolutionary Computation: Advanced Algorithms and Operators, Institute of Physics, London, 2000
7. X. Yao, An overview of evolutionary computation, Chinese J. of Advanced Software Research, 1996, vol. 3, No 1, pp. 1- 18
8. A. Oppenheim, R. Shafer, Cyfrowe przetwarzanie sygnałów; WKiŁ Warszawa 1979
9. M. Craig, Zarys cyfrowego przetwarzania, Warszawa 1999

ACKNOWLEDGMENT

The work has been supported by Polish Ministry of Science (KBN) as the research project.