Power Emitted Inside a Conducting Cylinder Placed in Longitudinal Uniform Magnetic Field of a Character an Attenuated Sinusoid

Zygmunt Piątek, Bernard Baron, Marian Pasko, Borys Borowik

Abstract **—In the paper we determine transient electromagnetic field in a conducting cylinder placed in external magnetic field having the character of an attenuated sinusoid through the solution of Bessel equation in cylindrical co-ordinates using Laplace integral transformation. Then, we use Poynting theorem to determine the superficial density of the instantaneous power flux diffused into the cylindrical charge and the volume density of the instantaneous power converted into heat inside this charge.**

I. Introduction

Magnetic field of a character of an attenuated sinusoid is used in metal forming and consists of applying energy of impulse magnetic field to the process. The impulse of the field is obtained due to the flow of impulse current, generated by a high-current impulse generator, through an operating head (a solenoid or flat bobbin) [1, 2]. In the conducting cylinder forming the impulse magnetic field is external in relation to the cylinder and has one component along the Oz axis (Fig.1) and it is determined by the following formula

$$\boldsymbol{H}^{zew}(t) = \mathbf{1}_z H_z^{zew}(t), \qquad (1)$$

where the component of the magnetic field strength along the Oz axis

$$H_z^{zew}(t) = H_0\, \mathrm{e}^{-\eta t} \sin(\varpi t + \xi)\, \mathbf{1}(t), \qquad (1a)$$

where: H_0 - magnetic field amplitude when, attenuation in $\mathrm{A \cdot m^{-1}}$ is non-existent, ω - pulsation of proper oscillation of the system element being formed-operating head- capacitor bank in $\mathrm{rad \cdot s^{-1}}$, η - magnetic field attenuation coefficient in $\mathrm{s^{-1}}$, ξ initial phase of the magnetic field strength in rad, $\mathbf{1}(t)$ - Headviside unit step.

Typical average values of the above quantities appearing in the impulse magnetic field metal forming are: $\xi = 0$, $H_0 = 10^7\,\mathrm{A \cdot m^{-1}}$, $\eta = 5 \cdot 10^3\,\mathrm{s^{-1}}$, $\varpi = \pi \cdot 10^4\,\mathrm{rad \cdot s^{-1}}$.

In the impulse magnetic field metal forming besides the basic problem consisting of determining the space-time distribution of the pressure in the element being formed (having previously determined the volume density of electrodynamic forces) it is also important to determine the power emitted inside the metal. We will deal with this question after the determination of the space-time distribution of current density $\boldsymbol{J}(r,t)$ and of the magnetic field strength $\boldsymbol{H}^I(r,t)$ in the element being formed, i.e. of the electromagnetic field.

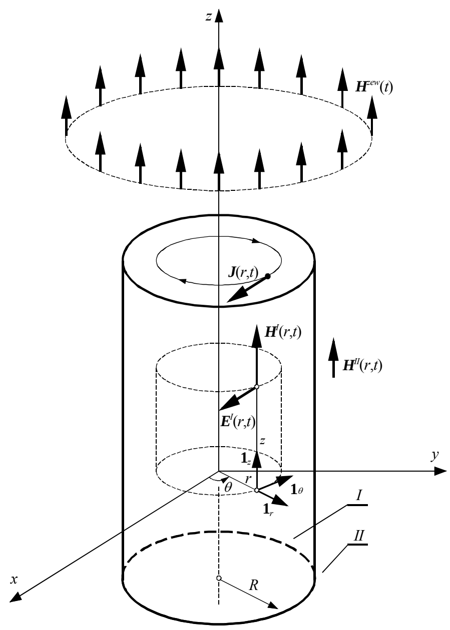

Fig. 1 Conducting cylinder in external longitudinal uniform magnetic field of a character of an attenuated sinusoid

II. Electromagnetic field

In the case of an infinitely long conducting cylinder placed in external longitudinal magnetic field (Fig.1) the values describing the electromagnetic field as for the symmetry of the system depend only on the r co-ordinate of the cylindrical co-ordinate system. Then, we deal here with a one-dimensional question with a constant magnetic permittivity of the cylinder

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$\mu = \mu_0$ and its constant conductivity γ. As the field $\boldsymbol{H}^{zew}(t)$ has got only one component along the Oz axis, form the second Maxwell equation $\mathbf{rot}\boldsymbol{E}^{zew}(r,t) = -\mu \dfrac{\partial \boldsymbol{H}^{zew}(t)}{\partial t}$, the electric field strength has also got one component along the axis Θ, i.e. $\boldsymbol{E}^{zew}(r,t) = -\mathbf{1}_\Theta E_\Theta^{zew}(r,t)$. So we have to deal with a question of the cylindrical wave cast on the lateral surface of the conducting cylinder.

In the genral case of a conductor of a chosen kind placed in alternating electromagnetic field some currents are bound to appear, as the total electric field cannot equal zero everywhere in the whole conductor. Those currents are called Foucault currents [5 ,6] and are determined by the current density vector $\boldsymbol{J}(r,t)$ - Fig.1. These currents generate the so-called return interaction magnetic field $\boldsymbol{H}^{oz}(r,t)$, which, in the system we are considering, has got one component along the z axis, thus $\boldsymbol{H}^{oz}(r,t) = \mathbf{1}_z H_z^{oz}(r,t)$. In papers [5, 6, 7, 8] it has been shown that this field equals zero. The zero value of the return interaction magnetic field in $r > R_2$ area results form the fact that the lines of the density of current $\underline{\boldsymbol{J}}(r)$ induced in the tubular charge are concentric circles of Oz axis– Fig.1. Then they do not generate any magnetic field outside the tube, as it is also the case with the current in the infinitely long solenoid. Then the total magnetic field in the considered area

$$\boldsymbol{H}^{II}(t) = \mathbf{1}_z H_z^{II}(t) = \mathbf{1}_z H_z^{zew}(t) = \mathbf{1}_z \operatorname{Im}\{\underline{H}_z^{zew}(t)\}, \quad (2)$$

where

$$\underline{H}_z^{zew}(t) = \underline{H}_0 \, \mathrm{e}^{-\eta t} \, \mathrm{e}^{\mathrm{j}\varpi t} \, \mathbf{1}(t), \quad (2a)$$

where the complex amplitude of the external magnetic field

$$\underline{H}_0 = H_0 \, \mathrm{e}^{\mathrm{j}\xi}. \quad (2b)$$

The required magnetic field strength $H_z^I(r,t)$ in the area of I ($0 \le r \le R$) is written as $H_z^I(r,t) = \operatorname{Im}\{\underline{H}_z^I(r,t)\}$, where $\underline{H}_z^I(r,t)$ is the complex magnetic field function of real variables r and t. This function fulfils the scalar wave equation in cylindrical co-ordinates

$$\frac{\partial^2 \underline{H}_z^I(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial \underline{H}_z^I(r,t)}{\partial r} - \mu\gamma \frac{\partial \underline{H}_z^I(r,t)}{\partial t} = 0. \quad (3)$$

For $r = R$ it has to be the case of the continuity of the magnetic field strength, i.e. we have the following boundary condition for the complex values:

$$\underline{H}_z^I(R,t) = \underline{H}_z^{\mathrm{zew}}(t). \quad (3a)$$

Moreover we assume a zero initial condition, i.e. for $t = 0$

$$\underline{H}_z^I(r,0) = 0. \quad (3b)$$

We solve equation (3) with the boundary condition (3a) applying Laplace integral transform. In order to do this let us denote by $\overline{H}_z^I(r,s)$ the Laplace transform complex function $\underline{H}_z^I(R,t)$ in relation to variable t, and then we perform the Laplace transformation of the following terms of the differential equation (3), taking into account the zero initial condition $\underline{H}_z^I(r,0) = 0$. Thus, we obtain the following equation [5, 6, 8]:

$$\frac{\partial^2 \overline{H}_z^I(r,s)}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{H}_z^I(r,s)}{\partial r} - s\mu\gamma\, \overline{H}_z^I(r,s) = 0 \quad (4)$$

with the boundary condition

$$\overline{H}_z^I(R,s) = \underline{H}_0 \frac{1}{s - s_0}, \quad (4a)$$

where

$$s_0 = -\eta + \mathrm{j}\,\varpi. \quad (4b)$$

Equation (4) is the Bessel equation of zero order of variable *r*, whose solution is the function [5, 6, 7, 8]

$$\overline{H}_z^I(r,s) = \underline{H}_0 \frac{I_0(\sqrt{s}\sqrt{\mu\gamma r})}{(s - s_0)\, I_0(\sqrt{s}\sqrt{\mu\gamma R})}, \quad (5)$$

where $I_0(\sqrt{s}\sqrt{\mu\gamma r})$ is the modified Bessela function of the complex variable $\sqrt{s}\sqrt{\mu\gamma}r$ of first kind and zero order. The function zeros of the denominator in formula (5) are $s = s_0$ and

$$s = s_k = -\sigma_k = -\frac{x_k^2}{\mu\gamma R^2} < 0, \quad (5a)$$

where

$$x_k \cong \varphi_k + \frac{1}{8\varphi_k} - \frac{124}{3(8\varphi_k)^3} + \frac{120928}{158(8\varphi_k)^5} - \dots, \quad (5b)$$

where

$$\varphi_k = (k - \frac{1}{4})\pi, \quad (k = 1, 2, 3, \dots). \quad (5c)$$

Then to calculate the original $\underline{H}_z^I(r,t)$ of the operational function $\overline{H}_z^I(r,s)$ we use the distribution theorem, obtaining [5, 6, 7, 8]

$$\underline{H}_z^I(r,t) = \left[\underline{H}_{z,0}^I(r,t) + \sum_{k=1}^{\infty} \underline{H}_{z,k}^I(r,t)\right]\mathbf{1}(t), \quad (6)$$

where $\underline{H}_{z,0}^I(r,t)$ is the original of function (5) in the pole $s = s_0$ ($k = 0$), while $\underline{H}_{z,k}^I(r,t)$ is the original of this function in the pole $s = s_k$ (k = 1, 2, 3,...). These originals are given by the following formulas:

$$\underline{H}^{I}_{z,0}(r,t) = \underline{H}_0 \frac{I_0(\underline{\Gamma}\, r)}{I_0(\underline{\Gamma}\, R)} \mathrm{e}^{-\eta t}\, \mathrm{e}^{\mathrm{j}\varpi t}, \qquad (6a)$$

and

$$\underline{H}^{I}_{z,k}(r,t) = \underline{H}_0 \frac{I_0(-\mathrm{j}x_k \frac{r}{R})}{\underline{A}_k(x_k) I_1(-\mathrm{j}x_k)} \exp[-\frac{x_k^2}{\mu\gamma R^2} t], \qquad (6b)$$

where $I_1(-\mathrm{j}\, x_k)$ is the modified Bessel function of first kind and first order.

The complex propagation constant

$$\underline{\Gamma} = \sqrt{-\eta\, \mu\, \gamma + \mathrm{j}\, \varpi\, \mu\, \gamma} = \Gamma\, \mathrm{e}^{\mathrm{j}\, \varphi}, \qquad (7)$$

whose module

$$\Gamma = \sqrt{\varpi \mu\, \gamma \sqrt{1 + (\frac{\eta}{\omega})^2}} = \kappa\, k \qquad (7a)$$

and the argument

$$\varphi = \frac{1}{2} \mathrm{arc\ tg}(-\frac{\varpi}{\eta}) = \frac{\pi}{4} + \frac{1}{2} \mathrm{arc\ tg} \frac{\eta}{\varpi}, \qquad (7b)$$

where

$$\kappa = \sqrt{2 \sqrt{1 + (\frac{\eta}{\omega})^2}} \qquad (7c)$$

and the coefficient

$$k = \sqrt{\frac{\varpi\, \mu\, \gamma}{2}}, \qquad (7d)$$

whose inverse is the depth of the diffusion of the wave inside the well conducting medium and it is

$$\delta = \frac{1}{k} = \sqrt{\frac{2}{\varpi\, \mu\, \gamma}}. \qquad (7e)$$

The complex propagation constant can also be written as

$$\underline{\Gamma} = k \sqrt{2\,(\mathrm{j} - \frac{\eta}{\varpi})} = \underline{\kappa}\, k, \qquad (7f)$$

where

$$\underline{\kappa} = \sqrt{2\,(\mathrm{j} - \frac{\eta}{\varpi})} = \kappa\, \mathrm{e}^{\mathrm{j}\varphi}. \qquad (7f)$$

Then the complex constant

$$\underline{A}_k(x_k) = \frac{1}{2x_k}[2k^2R^2 + \mathrm{j}\,(2k^2R^2 \frac{\eta}{\varpi} - x_k^2)] = \\ = A_k(x_k) \exp[\mathrm{j}\alpha_k(x_k)], \qquad (8)$$

where its module

$$A_k(x_k) = \frac{1}{2x_k} \sqrt{(2k^2R^2)^2 + (2k^2R^2 \frac{\eta}{\varpi} - x_k^2)^2} \qquad (8a)$$

and its argument

$$\alpha_k(x_k) = \mathrm{arc\ tg} \frac{2k^2R^2 \frac{\eta}{\varpi} - x_k^2}{2k^2R^2}. \qquad (8b)$$

The exponential form of the Bessel function appearing in formulas (6a) and (6b)

$$\left.\begin{aligned} &I_0(\underline{\Gamma}\, r) = M_0(\underline{\Gamma}\, r) \exp[\mathrm{j}\, \beta_0(\underline{\Gamma}\, r)], \\ &I_0(\underline{\Gamma}\, R) = M_0(\underline{\Gamma}\, R) \exp[\mathrm{j}\, \beta_0(\underline{\Gamma}\, R)], \\ &I_0(\text{-j}\, x_k \frac{r}{R}) = M_{0,k}(\text{-j}\, x_k \frac{r}{R}) \exp[\mathrm{j}\, \beta_{0,k}(\text{-j}\, x_k \frac{r}{R})], \\ &I_1(\text{-j}\, x_k) = M_{1,k}(\text{-j}\, x_k) \exp[\mathrm{j}\, \beta_{1,k}(\text{-j}\, x_k)]. \end{aligned}\right\} \qquad (9)$$

lets us to write the functions (6a) and (6b) in real forms, i.e. as real functions of variable r of the cylindrical co-ordinate system and of time t. We obtain respectively

$$H^{I}_{z,0}(r,t) = H_0 \frac{M_0(\underline{\Gamma}\, r)}{M_0(\underline{\Gamma}\, R)} \mathrm{e}^{-\eta t} \cdot \\ \cdot \sin[\varpi\, t + \beta_0(\underline{\Gamma}\, r) - \beta_0(\underline{\Gamma}\, R) + \xi], \qquad (10)$$

and

$$H^{I}_{z,k}(r,t) = H_0 \frac{M_0(\text{-j}x_k \frac{r}{R})}{A_k(x_k) M_1(\text{-j}x_k)} \exp[-\frac{x_k^2}{\mu\gamma R^2} t] \cdot \\ \cdot \sin[\beta_{0,k}(\text{-j}x_k \frac{r}{R}) - \beta_{1,k}(\text{-j}x_k) - \alpha_k(x_k) + \xi] \qquad (10a)$$

Finally the magnetic field strength in a conducting cylinder placed in external longitudinal magnetic field of a character of an attenuated sinusoid has the following form

$$H^{I}_{z}(r,t) = \left[H^{I}_{z,0}(r,t) + \sum_{k=1}^{\infty} H^{I}_{z,k}(r,t) \right] \mathbf{1}(t). \qquad (10b)$$

It is convenient to perform the analysis of the electromagnetic field in relative units. That is why we introduce the variable x corresponding to the variable r of the cylindrical co-ordinate system, as

$$x = \frac{r}{R}, \qquad 0 \le x \le 1. \qquad (11)$$

The frequency of the sinusoidal external magnetic field and the conductivity of the charge with regard to its external radius are taken into account through the coefficient $\alpha = \frac{R}{\delta} = k\, R$. Thus, we have: $\underline{\Gamma}\, r = \underline{\kappa}\, k\, r = \underline{\kappa}\, \alpha\, x$, $\underline{\Gamma}\, R = \underline{\kappa}\, k\, R = \underline{\kappa}\, \alpha$.

The magnetic field is then described by the following formulas:

$$H_z^I(x,t)=\left[H_{z,0}^I(x,t)+\sum_{k=1}^{\infty}H_{z,k}^I(x,t)\right]\mathbf{1}(t)\ . \tag{12}$$

where

$$H_{z,0}^I(x,t)=H_0\,\frac{M_0(\underline{\kappa}\,\alpha\,x)}{M_0(\underline{\kappa}\,\alpha)}\,\mathrm{e}^{-\eta t}\cdot\sin[\varpi\,t+\beta_0(\underline{\kappa}\,\alpha\,x)-\beta_0(\underline{\kappa}\,\alpha)+\xi]\ , \tag{12a}$$

and

$$H_{z,k}^I(x,t)=H_0\,\frac{M_0(\text{-j}x_k x)}{A_k(x_k)\,M_1(\text{-j}x_k)}\,\exp[-\frac{\varpi x_k^2}{2\alpha^2}t]\cdot\sin[\beta_{0,k}(\text{-j}x_k x)-\beta_{1,k}(\text{-j}x_k)-\alpha_k(x_k)+\xi] \tag{12b}$$

In the conducting cylinder the complex magnetic field strength $\underline{\boldsymbol{H}}^I(r,t)=\mathbf{1}_z\,\underline{H}_z^I(r,t)$ is connected with the complex current density $\underline{\boldsymbol{J}}(r,t)=\mathbf{1}_\Theta\,\underline{J}_\Theta(r,t)$ by the first Maxwell equation $\mathbf{rot}\,\underline{\boldsymbol{H}}^I(r,t)=\underline{\boldsymbol{J}}(r,t)$, from which, taking into account the fact that the above vectors have got only one component, we obtain

$$J_\Theta(x,t)=\left[J_{\Theta,0}(x,t)+\sum_{k=1}^{\infty}J_{\Theta,k}(x,t)\right]\mathbf{1}(t)\ , \tag{13}$$

where

$$J_{\Theta,0}(x,t)=-H_0\,\frac{M_1(\underline{\kappa}\,\alpha\,x)}{M_0(\underline{\kappa}\,\alpha)}\,\Gamma\,\mathrm{e}^{-\eta t}\cdot\sin[\varpi\,t+\beta_1(\underline{\kappa}\,\alpha\,x)-\beta_0(\underline{\kappa}\,\alpha)+\varphi+\xi] \tag{13a}$$

where $M_1(\underline{\kappa}\,\alpha\,x)$ and $\beta_1(\underline{\kappa}\,\alpha\,x)$ are respectively the module and the argument of the modified Bessel function of first kind and first order $I_1(\underline{\kappa}\,\alpha\,x)=M_1(\underline{\kappa}\,\alpha\,x)\,\mathrm{e}^{\mathrm{j}\beta_1(\underline{\kappa}\,\alpha\,x)}$, and

$$J_{\Theta,k}(x,t)=H_0\,\frac{M_1(\text{-j}x_k x)}{A_k(x_k)M_1(\text{-j}x_k)}\,\frac{k}{\alpha}\,x_k\exp[-\frac{\varpi x_k^2}{2\alpha^2}t]\cdot\cos[\beta_{1,k}(\text{-j}x_k x)-\beta_{1,k}(\text{-j}x_k)-\alpha_k(x_k)+\xi] \tag{13b}$$

III. SUPERFICIAL DENSITY OF THE INSTANTANEOUS POWER FLUX

According to the Poynting theorem [3, 9] the superficial density of the instantaneous power flux is defined by the following formula:

$$\boldsymbol{P}(x,t)=\frac{1}{\gamma}\boldsymbol{J}(x,t)\times\boldsymbol{H}^I(x,t)=P_r(x,t)\,(-\mathbf{1}_r)\ . \tag{14}$$

Substitution of (15) and (16) into this formula yields

$$P_r(x,t)=-\frac{1}{\gamma}\left[J_{\Theta,0}(x,t)+\sum_{l=1}^{\infty}J_{\Theta,l}(x,t)\right]\left[H_{z,0}^I(x,t)+\sum_{k=1}^{\infty}H_{z,k}^I(x,t)\right]\mathbf{1}(t)\ , \tag{14a}$$

where the fixed component and the transient component of the current density are given respectively by the formulas (13a) and (13b), while for the magnetic field by the formulas (12a) and (12b).

The influence of the parameter α on the distribution of the superficial density of the power flux inside the circular charge is shown in Fig. 2 at $t = T/4$, i.e. for the instantaneous value of the external magnetic field equal to its amplitude. This graph has been worked out for relative values, that is to say in relation to the value of the power flux on the surface of the cylindrical charge, i.e. as the coefficient $P_1=\dfrac{P_r(x,t=T/4)}{P_r(x=1,t=T/4)}$.

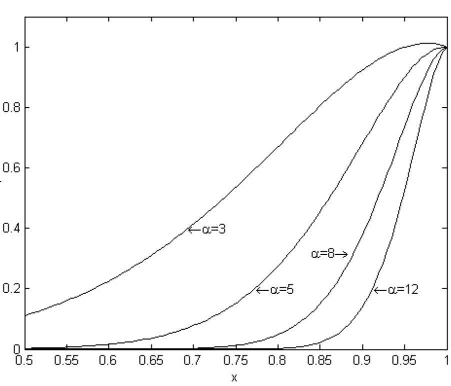

Fig.2. Diffusion and attenuation of the superficial density of the power flux inside a circular charge placed in external uniform magnetic field of a character of an attenuated sinusoid at $t = T/4$, $\omega = \pi\cdot10^4$ rad·s^{-1}, $\eta = 5\cdot10^3$ s^{-1}, $\gamma = 58\cdot10^6$ S·m^{-1}, $\xi = 0$: 1 – α = 3, 2 – α = 6, 3 – α = 12

The time-space distribution is shown in Fig.3. This graph has been worked out for relative values, that is to say in relation to $\dfrac{H_0^2\,\Gamma}{\gamma}$, i.e. as the coefficient

$$P_2=\frac{\gamma}{H_0^2\,\Gamma}P_r(x,t)\cdot$$

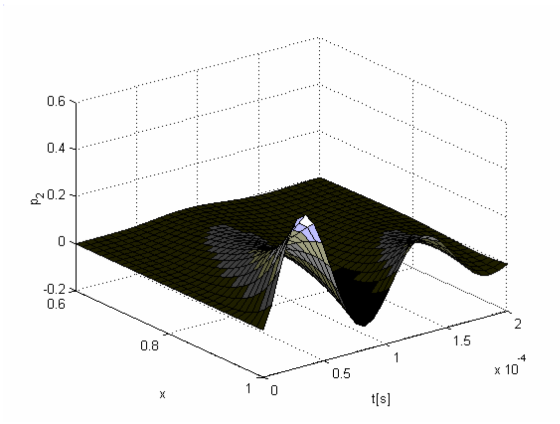

Fig.3. Time-space distribution of the power flux inside a circular charge placed in external uniform magnetic field of a character of an attenuated sinusoid; $\alpha = 5$, $\omega = \pi\cdot 10^4$ rad·s^{-1}, $\eta = 5\cdot 10^3$ s^{-1}, , $\xi = 0$, $\gamma = 58\cdot 10^6$ S·m^{-1}

The determined superficial density of the power flux inside the cylindrical charge allows us to determine the energy supplied to the charge in the time interval of $< 0, t_0 >$. It is determined from the instantaneous power per unit length of the conductor, diffused through its lateral surface.

$$p(t) = 2\pi R P_r(x=1,t)\,\mathbf{1}(t) = \\ = -2\pi\frac{\alpha}{k\gamma}\left[J_{\Theta,0}(x=1,t) + \sum_{l=1}^{\infty} J_{\Theta,l}(x=1,t)\right]\cdot \\ \left[H^{I}_{z,0}(x=1,t) + \sum_{k=1}^{\infty} H^{I}_{z,k}(x=1,t)\right]\mathbf{1}(t) \tag{15}$$

integration of this power in relation to the time from 0 to t_0 , i.e.

$$W = \int_0^{t_0} p(t)dt\,. \tag{16}$$

IV. VOLUME DENSITY OF THE INSTANTANEOUS POWER

In order to determine the temperature distribution inside the charge one has to define the so called external heat sources, which are defined by the volume density of the power converted into heat. According to Poynting theorem the instantaneous volume density of the power converted into heat in area V in $[\mathrm{W}\cdot\mathrm{m}^{-3}]$

$$p_{cal}(r,\Theta,z,t) = \frac{1}{\gamma}J^2(r,\Theta,z,t) \tag{17}$$

and then the instantaneous power in [W]

$$p_{cal}(t) = \int_V p_{cal}(r,\Theta,z,t)\,\mathrm{d}V = \frac{1}{\gamma}\int_V J^2(r,\Theta,z,t)\,\mathrm{d}V. \tag{18}$$

In the case of the current density depending only on variable r of cylindrical co-ordinate system, i.e. for $J = J_\Theta(r,t)$ this power is given by the formula:

$$p_{cal}(t) = \frac{1}{\gamma}\int_0^l\int_0^{2\pi}\int_0^R J^2_\Theta(r,t)\,r\,\mathrm{d}r\,\mathrm{d}\Theta\,\mathrm{d}z = \\ = \frac{2\pi l}{\gamma}\int_0^R J^2_\Theta(r,t)\,r\,\mathrm{d}r \tag{19}$$

Substitution of the variable $x = \dfrac{r}{R} \to r = xR \to \mathrm{d}r = R\,\mathrm{d}x$, where $0 \le x \le 1$ yields respectively the instantaneous volume density of the power

$$p_{cal}(x,t) = \frac{1}{\gamma}J^2_\Theta(x,t) \tag{20}$$

as well as the instantaneous power converted into heat in area V

$$p_{cal}(t) = \int_V p_{cal}(x,t)\,\mathrm{d}V = \frac{2\pi l R^2}{\gamma}\int_0^1 J^2_\Theta(x,t)\,x\,\mathrm{d}x = \\ = \frac{2\pi l\alpha^2}{\gamma k^2}\int_0^1 J^2_\Theta(x,t)\,x\,\mathrm{d}x \tag{21}$$

The time-space distribution of the volume density of the loses of calorific power is shown in Fig. 4. The graph is worked out for relative values, i.e. in relation to $\dfrac{H_0^2 k^2}{\gamma}$, i.e. in the form of function

$$p_3(x,t) = \frac{\gamma}{H_0^2 k^2}p_{cal}(x,t)\,.$$

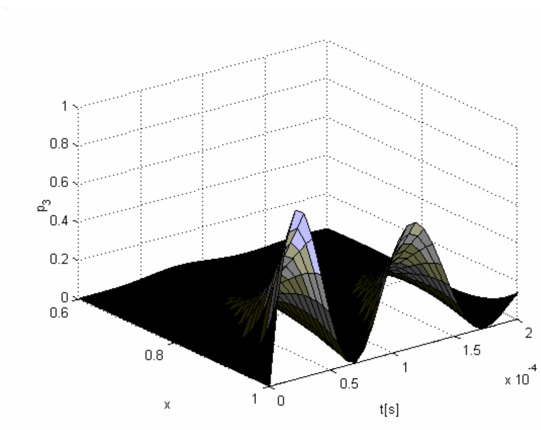

Fig.4. Time-space distribution of the volume density of heat loses inside a cylindrical charge placed in external uniform magnetic field of a character of an attenuated sinusoid j; $\alpha = 5$, $\omega = \pi\cdot 10^4$ rad·s^{-1}, $\eta = 5\cdot 10^3$ s^{-1}, $\gamma = 58\cdot 10^6$ S·m^{-1}, $\xi = 0$

The curve of the instantaneous power converted into heat inside a cylindrical charge is shown in Fig..5. The graph is worked out for relative values, that is to say in relation to $\dfrac{\sqrt{2}\,\pi l\alpha H_0^2}{\gamma}$, i.e. in the form of function

$$p_4(t) = \frac{\gamma}{\sqrt{2}\,\pi l\alpha H_0^2}p_{cal}(t)\,.$$

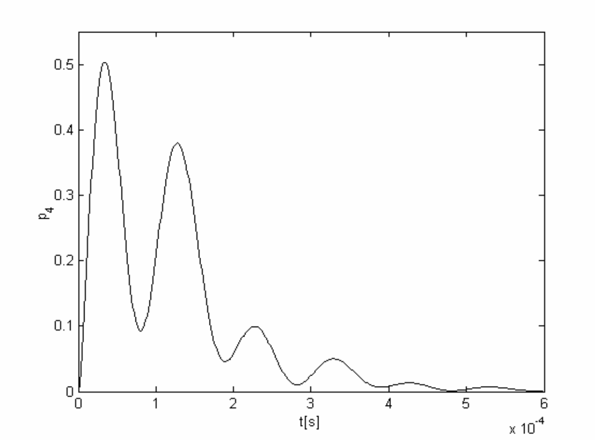

Fig.5. Curve of the instantaneous power converted into heat inside a cylindrical charge placed in external uniform magnetic field of a character of an attenuated sinusoid; $\alpha = 5$, $\omega = \pi\cdot 10^4$ rad·s^{-1}, $\eta = 5\cdot 10^3$ s^{-1}, $\gamma = 58\cdot 10^6$ S·m^{-1}, $\xi = 0$

Energy converted into heat during the time $0 \le t \le t_0$ is given by the formula

$$W_{cal} = \int_0^{t_0} p_{cal}(t)\,\mathrm{d}t\,. \tag{22}$$

For the practical purposes $\eta = 5\cdot 10^3\,\mathrm{s}^{-1}$ and $\varpi = \pi\cdot 10^4\,\mathrm{rad}\cdot\mathrm{s}^{-1}$ which means that the relation $\dfrac{H_z^{zew}(t = \frac{T}{4} + 5T)}{H_z^{zew}(t = \frac{T}{4})}$ does not surpass 1%. In order to determine the energy converted into heat inside a cylindrical charge we can then (see also Fig. 5) quite acurately assume the time $t_0 = 5T$.

V. Conclusion

For the case of the fixed sinusoidal electromagnetic field ($\eta = 0$) in the electric and magnetic field the transient components disappear, i.e. the magnetic field is given by the formula

$$H_z^I(x,t) = H_{z,0}^I(x,t)\,\mathbf{1}(t) = H_0\,\frac{M_0(\underline{\kappa}\,\alpha\,x)}{M_0(\underline{\kappa}\,\alpha)}\cdot \\ \cdot\sin[\varpi t + \beta_0(\underline{\kappa}\,\alpha\,x) - \beta_0(\underline{\kappa}\,\alpha) + \xi]\,\mathbf{1}(t) \tag{23}$$

while the current density

$$J_\Theta(x,t) = J_{\Theta,0}(x,t)\,\mathbf{1}(t) = -H_0\,\frac{M_1(\underline{\kappa}\,\alpha\,x)}{M_0(\underline{\kappa}\,\alpha)}\,\Gamma\cdot \\ \cdot\sin[\varpi t + \beta_1(\underline{\kappa}\,\alpha\,x) - \beta_0(\underline{\kappa}\,\alpha) + \varphi + \xi]\,\mathbf{1}(t) \tag{24}$$

in which $\underline{\kappa} = \sqrt{2\,\mathrm{j}}$, $\Gamma = \sqrt{\varpi\mu\gamma} = \sqrt{2}\,k$ and $\varphi = \frac{\pi}{4}$.

Then the superficial density of the instantaneous power is given by the formula

$$P_r(x,t) = -\frac{1}{\gamma}\,J_{\Theta,0}(x,t)\,H_{z,0}^I(x,t)\,\mathbf{1}(t) = \\ = \sqrt{\frac{\varpi\mu}{\gamma}}\,\frac{H_0^2}{2}\,\frac{M_0(\sqrt{2\mathrm{j}}\,\alpha\,x)\,M_1(\sqrt{2\mathrm{j}}\,\alpha\,x)}{M_0^2(\sqrt{2\mathrm{j}}\,\alpha)}\cdot \\ \cdot\left\{\begin{array}{l}\cos[\beta_1(\sqrt{2\mathrm{j}}\,\alpha\,x) - \beta_0(\sqrt{2\mathrm{j}}\,\alpha\,x) + \frac{\pi}{4}] - \\ -\cos[2\varpi t + \beta_1(\sqrt{2\mathrm{j}}\,\alpha\,x) + \\ + \beta_0(\sqrt{2\mathrm{j}}\,\alpha\,x) - 2\beta_0(\sqrt{2\mathrm{j}}\,\alpha) + \frac{\pi}{4} + 2\xi]\end{array}\right\}\mathbf{1}(t) \tag{25}$$

Instantaneous power per unit length of the circular charge diffused through its lateral surface

$$p(t) = 2\pi R\,P_r(x=1,t) = \\ = -2\pi\frac{R}{\gamma}\,J_{\Theta,0}(x=1,t)\,H_{z,0}^I(x=1,t)\,\mathbf{1}(t) \tag{26}$$

Then the active power

$$P = \frac{1}{T}\int_0^T p(t)\,\mathrm{d}t = 2\pi R\sqrt{\frac{\varpi\mu}{\gamma}}\,\frac{H_0^2}{2}\,\frac{M_1(\sqrt{2\mathrm{j}}\,\alpha)}{M_0(\sqrt{2\mathrm{j}}\,\alpha)}\cdot \\ \cdot\cos[\beta_1(\sqrt{2\mathrm{j}}\,\alpha) - \beta_0(\sqrt{2\mathrm{j}}\,\alpha) + \frac{\pi}{4}] \tag{27}$$

which is, after the suitable modifications, the equivalent of the solution obtained by M. Krakowski in [3] – formula (9.111), p. 200.

The determined electromagnetic field and the superficial and volume density of the power can be used to define the substitute parameters of the system working head-charge during the process of metal forming with impulse magnetic field. The results presented here, describing the volume density of the power converted into heat inside a cylindrical charge determine the so-called external heat sources and can be used to describe the temperature field in the charge being formed both in transient and fixed states of the electromagnetic field.

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