Dynamic Characteristic of Aluminum Sphere Levitating in Electromagnetic Field Respecting its Induction Heating

Ivo Doležel, Pavel Karban, Martin Mach, Ladislav Musil, Bohuš Ulrych

***Abstract* — The paper deals with numerical solution of electromagnetic-thermo-mechanical transient associated with levitation heating of a nonferromagnetic sphere in harmonic electromagnetic field. Investigated is the period during which the body gets from the starting position to the final levitating position in the inductor that is, however, influenced by its next heating. The problem is solved in quasi-coupled formulation. Theoretical considerations are illustrated on an example whose results are discussed.**

I. INTRODUCTION

ELECTRODYNAMIC levitation of solid electrically conductive bodies represents the basis of a number of modern technologies. One of them is, for example, levitation melting of metals in an inert atmosphere. The basic advantage of this technology consists in the fact that the processed metal is not polluted by various impurities that could (when using classic technologies) penetrate into it due to its direct contact with the crucible wall. The result is a superclean pure metal or alloy, intended mostly for various medical, space or other advanced applications.

The levitating system itself consists of one or more mutually connected or separated coils. Its arrangement may differ from one case to another (shapes of the particular coils can be cylindrical, conical or even more sophisticated). In specific applications, the basic coils may be placed in transversal magnetic field produced by supplementary coils that provides stabilizing rotation of the processed workpiece etc.

Design and optimization of the device and evaluation of the complete process require, however, mathematical and computer modeling that provides sufficient and reliable information about its characteristics and overall efficiency.

The paper deals only with the first part of the process - lifting the workpiece (an aluminum sphere) to the final position and its heating. This task represents a coupled electromagnetic-thermo-mechanical transient problem that is solved in a quasi-coupled formulation.

II. FORMULATION OF THE PROBLEM

Consider the simplest possible axi-symmetric levitating system depicted in Fig. 1 in a cylindrical coordinate system r, z. The system consists of the field coil **1** (that carries harmonic current of amplitude I and frequency f) and processed body **2** (well electrically conductive sphere)**.**

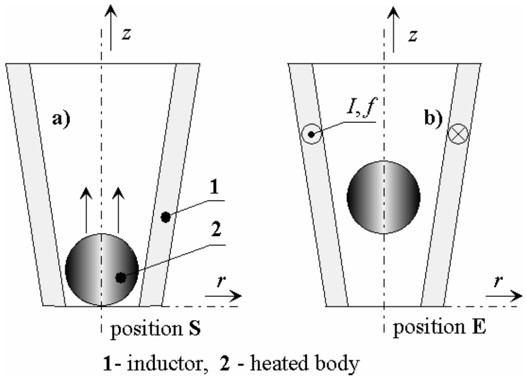

Fig. 1. The investigated arrangement

At the time $t = 0$ the inductor **1** is connected to the source of harmonic current (I, f) and starts producing harmonic magnetic field. This field induces eddy currents $\boldsymbol{J}_{\mathrm{eddy}}$ in the body **2**. Interaction between the magnetic field and eddy currents induced in the body then produces the Lorentz forces that make it move up from the starting position **S** (Fig. 1a) in the direction of the arrows. At the same time the body starts to be heated by the Joule losses. After the transient taking time t_1 the body reaches the end position **E** where the Lorentz forces acting on it are in balance with its weight (Fig. 1b).

The transient itself depends on a lot of parameters and its time evolution can be quite a complicated function (but mostly with character of slowly damped oscillations).

Growing temperature of the inductor and, particularly, processed body affects the physical properties of the system, among others electrical conductivity of its parts. Its variation influences distribution of magnetic field and, consequently, the position **E**. On the other hand, these variations are relatively small and in most cases may be neglected.

It is necessary to determine the dynamic characteristic of the device, i.e. the dependence of the position of the sphere on time, and describe consequent temperature rise. Respected should be both influence of geometry of the coil and eventual temperature variations as far as they are important.

III. MATHEMATICAL MODEL AND ITS SOLUTION

The mathematical model of the problem generally consists of two partial differential equations describing

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distribution of nonstationary electromagnetic and temperature fields in the system and one nonlinear ordinary differential equation describing the motion of the sphere.

A. Electromagnetic field

Its definition area of the task (in the axisymmetric arrangement) is depicted in Fig. 2. Line ABCD is the artificial boundary of the arrangement that represents the infinity (position of this boundary follows from several preliminary computations that show when the field distribution near the inductor **1** depends no longer on its distance). The investigated domain contains four subregions $\Omega_1, \ldots, \Omega_4$ with different physical parameters.

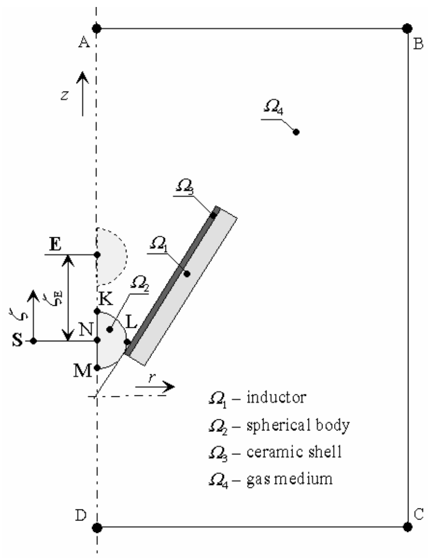

Fig. 2. Definition area of the task

As the system is linear, the electromagnetic field distribution may be described by Helmholtz' equation for the phasor of vector potential $\boldsymbol{A}$. Because the field is considered axisymmetric (so that the vector potential as well as the densities of both field and eddy currents have only one nonzero component in tangential direction φ_0), this equation reads [1]

$$\frac{\partial^2 \underline{A}_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \underline{A}_\varphi}{\partial r} - \frac{\underline{A}_\varphi}{r^2} + \frac{\partial^2 \underline{A}_\varphi}{\partial z^2} + \underline{k}^2 \underline{A}_\varphi = -\mu_0 \underline{J}_{\mathrm{ext}\varphi}, \quad (1)$$

$$\underline{k}^2 = -\mathrm{j} \cdot \omega\gamma\mu_0$$

where γ denotes the electrical conductivity (which is generally a function of temperature *T*), $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic permeability of vacuum and ω the angular frequency of the field current I. The phasor of their density denoted as $\underline{J}_{\mathrm{ext}\varphi}$ is known (the current parameters are supposed to be constant in time).

The conditions along the boundary ABCDA read:

- AD - antisymmetry, $\underline{A}_\varphi = 0$,
- ABCD - a force line along which $\underline{A}_\varphi = \mathrm{const}$. The condition of continuity of the vector potential at points A and D implies that this constant is identically equal to zero.

The calculated distribution of the phasor of vector potential $\underline{A}_\varphi = \underline{A}_\varphi(r, z)$ then provides distribution of eddy currents $\underline{J}_{\mathrm{eddy}\varphi}$, specific Joule's losses w_J and average specific Lorentz' forces $\boldsymbol{f}_\mathrm{L}$ acting on the heated sphere. These quantities are described by relations [2], [3]

$$\underline{J}_{\mathrm{eddy}\varphi} = -\mathrm{j} \cdot \omega\gamma \underline{A}_\varphi, \quad (2)$$

$$w_\mathrm{J} = \frac{\underline{J}_{\mathrm{eddy}\varphi} \cdot \underline{J}^*_{\mathrm{eddy}\varphi}}{\gamma} \quad (3)$$

and

$$\boldsymbol{f}_\mathrm{L} = \boldsymbol{J}_\mathrm{eddy} \times \boldsymbol{B} \quad (4)$$

where $\underline{J}^*_{\mathrm{eddy}\varphi}$ is the complex conjugate to $\underline{J}_{\mathrm{eddy}\varphi}$. Now it is necessary to analyze the last expression for $\boldsymbol{f}_\mathrm{L}$. It can be shown that

$$\boldsymbol{f}_\mathrm{L} = \underline{\boldsymbol{J}}_\mathrm{eddy} \times \underline{\boldsymbol{B}}^* = \underline{\boldsymbol{J}}_\mathrm{eddy} \times \mathrm{rot}\, \underline{\boldsymbol{A}}^* = \mathbf{r}_0 f_{\mathrm{L}r} + \mathbf{z}_0 f_{\mathrm{L}z} = \mathbf{r}_0 \cdot \frac{\underline{J}_{\mathrm{eddy}\varphi}}{r} \cdot \frac{\partial}{\partial r}\left(r \underline{A}^*_\varphi\right) - \mathbf{z}_0 \cdot \underline{J}_{\mathrm{eddy}\varphi} \cdot \frac{\partial \underline{A}^*_\varphi}{\partial z}. \quad (5)$$

The total Lorentz' force $\boldsymbol{F}_\mathrm{L}$ acting on the sphere has only one component $F_{\mathrm{L}z}$ in the axial direction

$$F_{\mathrm{L}z} = \int_{V_1} f_{\mathrm{L}z} \cdot \mathrm{d}V = -\int_{V_1} \underline{J}_{\mathrm{eddy}\varphi} \cdot \frac{\partial \underline{A}^*_\varphi}{\partial z} \cdot \mathrm{d}V \quad (6)$$

where V_1 is the volume of the sphere.

B. Nonstationary temperature field

The nonstationary temperature field is calculated only in the heated sphere, i.e. in domain Ω_1 bounded by semicircle KLMNK. The basic Fourier-Kirchhoff equation generally describing its distribution reads [4]

$$\mathrm{div}\left(\lambda \cdot \mathrm{grad}\, T\right) = \rho c \cdot \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \mathrm{grad}\, T\right) - w_\mathrm{J} \quad (8)$$

where λ denotes the thermal conductivity, ρ the specific mass of the heated material, c its specific heat and $\boldsymbol{v}$ its velocity. In fact, velocity $\boldsymbol{v}$ of the sphere has only one component v (in direction z) so that

$$\boldsymbol{v} \cdot \mathrm{grad}\, T = v \cdot \frac{\partial T}{\partial z}.$$

The boundary conditions are (see Fig. 2):

- KNM – symmetry: $\dfrac{\partial T}{\partial r} = 0$,
- KLM – convection: $-\lambda \cdot \dfrac{\partial T}{\partial z} = \alpha\left(T - T_\mathrm{ext}\right)$

where α is the convective heat transfer coefficient and T_ext the known temperature of ambient medium. Radiation is not respected in this case even when its inclusion does not represent any problem.

C. Equation of the transient

Motion of the sphere during the transient is described by a nonlinear ordinary differential equation based on the balance of all forces acting on it. The equation reads

$$F_\mathrm{a} = F_{\mathrm{L}z} - F_\mathrm{g} - F_\mathrm{d} \quad (9)$$

where F_a is the accelerating force, F_{Lz} the total Lorentz force acting on the sphere given by (6), F_g its weight and F_d the drag force given by aerodynamic resistances. All these quantities depend either on the position of the sphere or on its derivatives (velocity v and acceleration a). Particular forces can be described as follows:

$$F_a = m \cdot \frac{dv}{dt} = \rho V \cdot \frac{dv}{dt} \tag{10}$$

where ρ is the specific mass of the sphere and V its volume,

$$F_g = \rho V \cdot g \tag{11}$$

and finally

$$F_d = c_x \cdot \rho S v^2 / 2, \tag{12}$$

S being the cross-section of the sphere, ρ density of the medium and c_x the aerodynamic coefficient.

IV. Illustrative example

For the particular arrangement depicted in Figs. 1, 2 and 3 it is necessary to find

- Current of amplitude I and frequency f that would ensure
  – movement of the levitated sphere from the starting position **S** (see Fig. 2) to final position **E** where $F_{Lz} = F_g$.
  – its consequent induction heating to average temperature $T_A = 650\,°C$,

Input data and computations

The basic dimensions of the system (see Fig. 3) are:

- $R_0 = 0.05$ m, $h = 0.1$ m, $r_1 = 0.04$ m,
- $l = 0.178$ m, $s_1 = 0.003$ m, $s_2 = 0.002$ m,
- $\beta = 30$, 45 and 60°, respectively.

The coil is made from a hollow copper (Cu 99) conductor (internal diameter 4 mm, external diameter 8 mm) cooled by water. Its arrangement and dimensions are depicted in Fig. 3. The number of its turns that are wound in two layers $N_c = 36$.

The principal physical parameters of Cu 99: electrical conductivity $\gamma_{Cu} = 5.7 \cdot 10^7$ S/m, $\mu_r = 1$.

Material of the sphere is aluminum Al 99.5 with average electrical conductivity $\gamma_{Al} = 1.9 \cdot 10^7$ S/m, $\lambda = 229$ W/mK, $\rho = 2700\,kg/m^3$, $c = 896$ J/kgK and $\mu_r = 1$. The mass of the sphere $m = 1.45560$ kg and its weight $F_g = 13.87$ N. The average convective coefficient of heat transfer $\alpha = 20\,W/m^2K$ and $T_{ext} = 20$ °C. Magnetic permeability of electrically nonconductive ceramic shell **3** of the field coil is equal to μ_0 .

Numerical solution of the problem was carried out by combination of professional FEM-based code QuickField [5] supplemented with a number of single-purpose user programs written in Matlab (Simulink) and Borland Delphi.

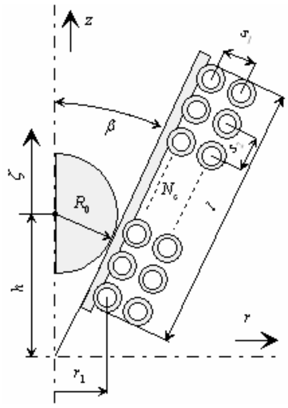

Fig. 3. The details of the coil

Selected results

It turned out that meshes with more than about 100000 elements provide sufficiently accurate (three valid digits) computation of magnetic field and consequent integral quantities such as energy, total Joule losses, total Lorentz force etc. The best parameters from the viewpoint of induction heating (see [6]) are $I = 1000$ A, $f = 5$ kHz, and $\beta = 30°$. Fig. 4 depicts the static characteristic of the system for these values.

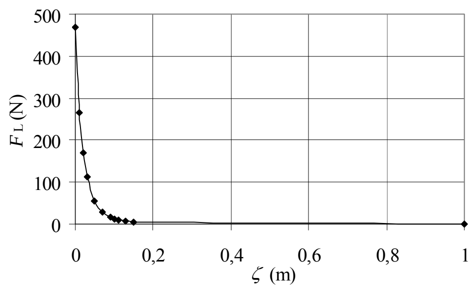

Fig. 5. Static characteristic of the system

The dynamic characteristic was first determined without damping (aerodynamic resistance of the gas medium was neglected). Now the characteristic depends only on the starting position **S** + h (the significance of symbol h follows from Fig. 3) and contains undamped oscillations (Figs. 5 and 6).

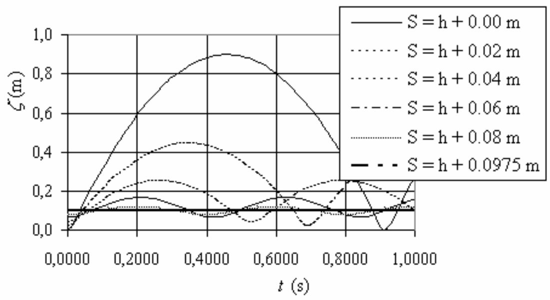

Fig. 5. Dynamic characteristic of the system ($\zeta = \zeta(t)$)

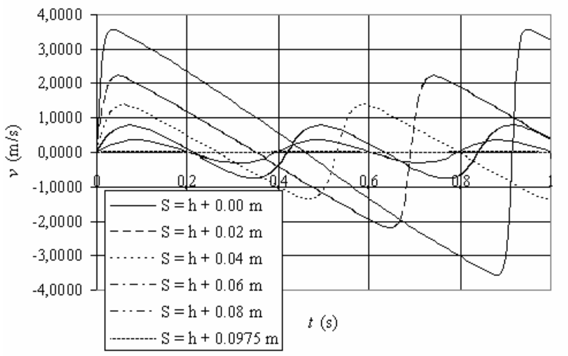

Fig. 6. Dynamic characteristic of the system ($v = v(t)$)

Finally Fig. 7 contains the dependencies of amplitudes and maximum velocities on the starting position.

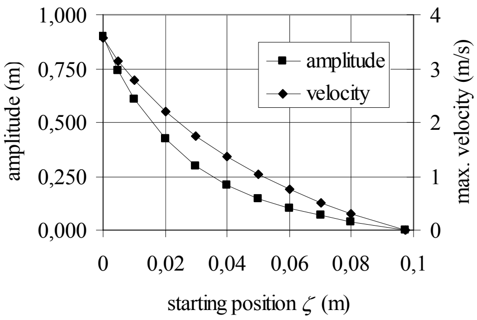

Fig. 7. Dependence of the maximum lift and velocity of the sphere on starting position ζ

In order to prevent the sphere from oscillations the starting position **S** should be as near as the end position **E** characterized by $\zeta_{\mathrm{E}} = 0.0975$ m.

As for damping due to aerodynamic resistances, we supposed that the process of heating is realized in argon. The coefficient of aerodynamic resistance c_{x} for free gas medium that is a function of the Reynolds number and estimated average velocity $v = 0.5$ m/s of movement was calculated according to [7]. Fig. 8 shows damping under normal atmospheric conditions and under a pressure.

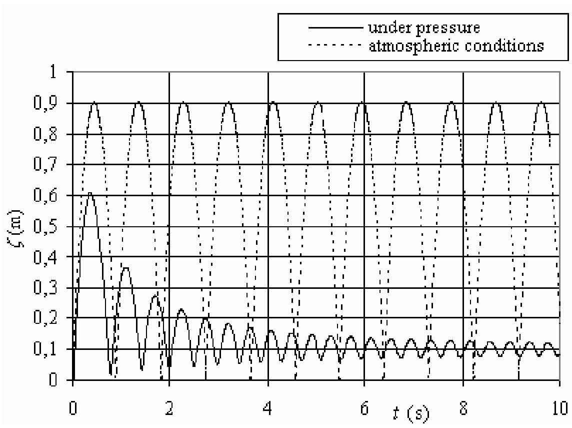

Fig. 8. Influence of damping (normal atmospheric conditions), starting position S = *h* (Fig. 3)

Even in case of the normal atmospheric conditions the starting position **S** should be as near as the end position **E.**

An example of the obtained temperature field for time $t = 580$ s, starting position $\zeta = 0.08$ m and optimal heating parameters $I = 1000$ A, $f = 5$ kHz and $\beta = 30°$ is depicted in Fig. 9. It can be seen that the temperature is distributed fully uniformly overall the sphere, which is caused mainly by its high thermal conductivity. Somewhat lower temperature along its surface is because of the heat convection into ambient medium.

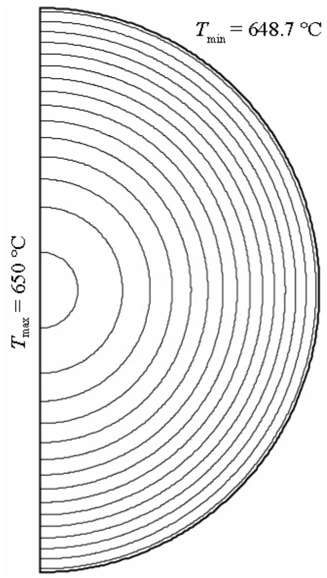

Fig. 9. Distribution of temperature field in the sphere ($I = 1000$ A, $f = 5$ kHz, $\beta = 30°$)

V. Conclusion

Damping of the body due to its aerodynamic resistance during the process of its lifting is very low, so that it is desirable to start it near the balance position. In such a case the oscillation are small, characterized by low velocities and, therefore, low variations of convective heat transfer coefficient α..

Next work in the field will be aimed at the investigation of various other shapes of the inductor in order to accelerate the process of heating and increase its efficiency

VI. Acknowledgment

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