The mathematical models of stomach polypean illnesses

Marcin Buczaj, Wiktor Pietrzyk

***Abstract* — The presented work proposes a mathematical model that describes polyps as stomach illnesses. It presents a practical application of the proposed model to measure polyp sizes. A method for determining polyp sizes on the basis of characteristic points plotted over the obtained video material has been also proposed.**

Authors proposed to use the prolate spheroidal coordinates and oblate spheroidal coordinates (η, θ, ψ) or the parabolic coordinates (μ, υ, ψ) for describe of share. The first coordinates system describe shares from spherical to segment. The second coordinates system describe figures from oblate spheroidal to spherical.

I. Introduction

Gastroscopic and endoscopic tests are non-invasive identification methods for the diagnosis of stomach illnesses. Polyps are rare, however dangerous, gastropathy leading to neoplastic lesions. The identification of pathology is performed on the basis of a video recording taken during tests on patients. The size and the type of an identified polyp determine the type of treatment methods.

The paper presents a mathematical that describes a gastropathy called a polyp. The research has been performed on images of gastric polyps taken st the Institute of Rural Medicine in Lublin, Poland and some images from the gastroenterologic atlas [1].

Polyps in a stomach occur relatively rarely an they are detected at 2-3% of patients. they are small changes of 1 – 2 cm diameter. They can lead to neoplastic changes [1].

Thanks to the fact that these lesions are rather of a regular shape, their size e.g. volume, can be easily estimated on the basis of a two dimensional image from an endoscope. This makes possible to describe these shapes by means of simple mathematical equations. Thanks to precise evaluation of volumes, proper treatment can be performed and complications can be avoided (e.g. haemorrhage) during removal (polypectomy).

II. Models

Thepolyps can be divided in three groups of shape:

a) polyps with visible the pedicle;
b) polyps with visible the basis;
c) polyps no visible either pedicle and base.

Authors are with the Department of Computer and Electrical Engineering, Faculty of Electrical Engineering and Computer Science, Lublin University of Technology, ul. Nadbystrzycka 38A, 20-618 Lublin, Poland, e-mail: mbuczaj*@elektron.pol.lublin.pl*

A. Polyps with a visible pedicle

The proposed model is based on a prolate spheroid in which the cross section perpendicular to z axis is semi circular.

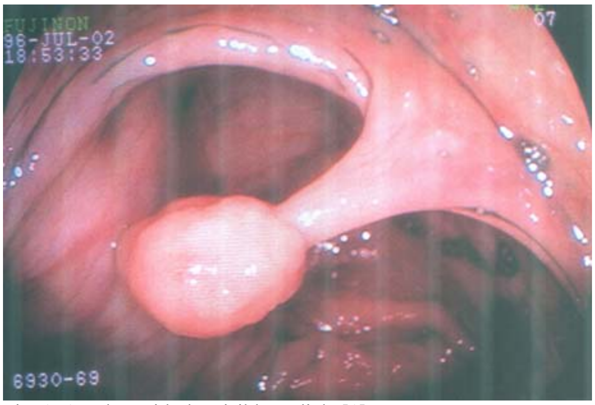

Fig. 1. A polyp with the visible pedicle [1]

The describing equation is the following:

$$\begin{cases} x = a \sinh \eta \sin \theta \cos \psi, \\ y = a \sinh \eta \sin \theta \sin \psi, \\ z = a \cosh \eta \cos \theta, \end{cases} \quad (1)$$

where:

$$0 \le \eta < \infty, \quad 0 \le \theta \le \pi, \quad 0 \le \psi \le 2\pi. \quad (2)$$

Assuming that $\eta = \text{const}$, the equation has the form of an ellipsoid. The model of this polyp type generated by MathCad is presented in Fig. 2.

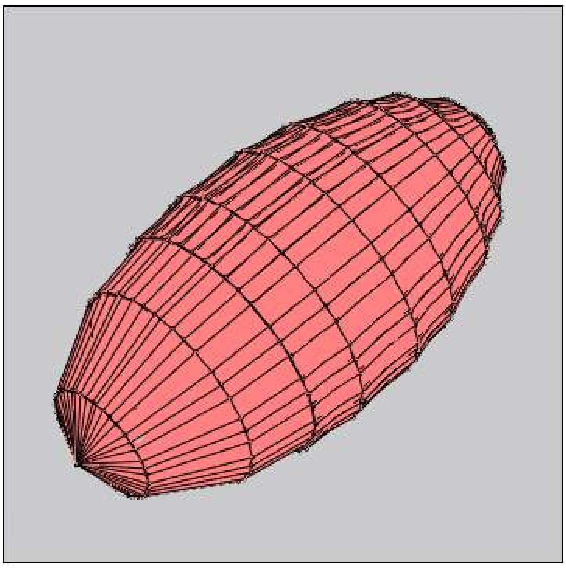

Fig.2. A polyp model with the visible pedicle

B. Polyps with the visible base

The mathematical model of a polyp with the visible base is based on prolate spheroid too but cut in z – axis.

Fig.3. A polyp with the visible base

The describing equation is the following:

$$\begin{cases} x = a\cosh\eta\sin\theta\cos\psi, \\ y = a\cosh\eta\sin\theta\sin\psi, \\ z = a\sinh\eta\cos\theta, \end{cases} \tag{3}$$

where:

$$\begin{aligned} &0 \le \eta < \infty, \\ &0 \le \theta \le \pi, \\ &0 \le \psi \le 2\pi. \end{aligned} \tag{4}$$

Assuming that $\eta =$ const, the equation (3) has the form of an ellipsoid.
The model of this polyp type generated by MathCad is presented in Fig. 4.

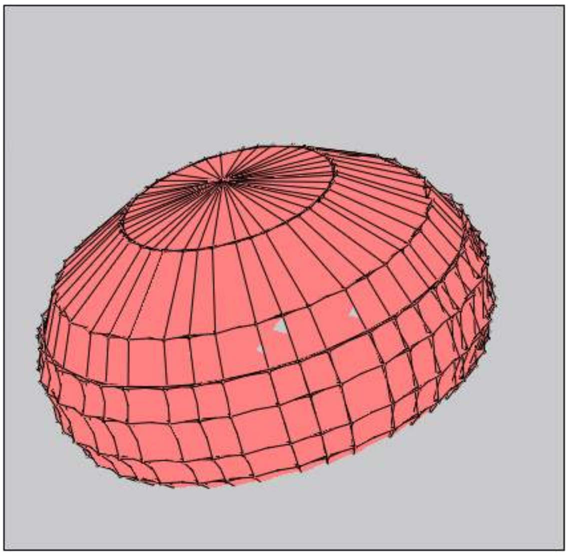

Fig. 4. A polyp model with the visible base

C. Polyps with no visible base and pedicle

The mathematical model of this type of a polyp is based on a paraboloid model.

Fig.5. A polyp with no visible base and pedicle [3]

The describing equation is the following:

$$\begin{cases} x = \mu\nu\cos\psi, \\ y = \mu\nu\sin\psi, \\ z = \frac{1}{2}\left(\mu^2 - \nu^2\right), \end{cases} \tag{5}$$

where:

$$\begin{aligned} &0 \le \mu < \infty, \\ &0 \le \nu \le \infty, \\ &0 \le \psi \le 2\pi. \end{aligned} \tag{6}$$

Assuming that $\nu =$ const, the equation (5) has the form of rotational paraboloid. The generated model of a polyp with no visible base and pedicle is presented in Fig. 6.

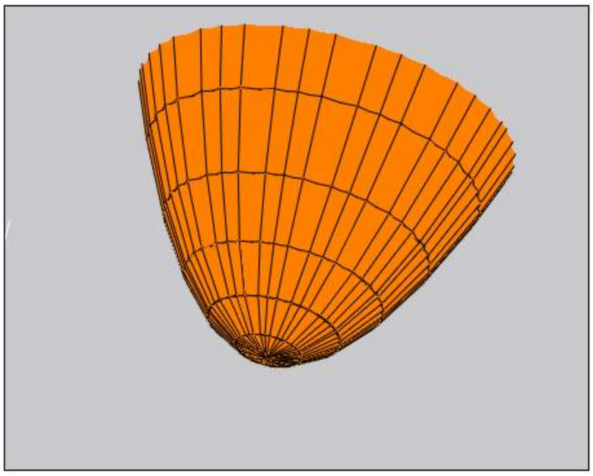

Fig. 6. The model of a polyp with no visible base and pedicle

III. Research Results

The presented mathematical models of different polyp types make possible to compose an algorithm to evaluate size (volume) of polyps on the basis of a few points introduced on the image from an endoscope camera. The effect of the performed research has been obtained in the form of computer software that enables fast determination of the size of the tested pathology. This procedure makes possible to shorten the time of analysis of the given video recording.

The presented models help determine the minimal number of points required to be introduced on analyzed images obtained from the endoscope to evaluate the size of the tested pathology. The characteristic points introduced on the image make possible to determine lengthy of particular distances. The required number of points for different polyps is given in Table 1.

TABLE I. POINTS REGUIRED TO DETERMINE POLIP SIZE

Polyp type	Point quantity
Visible pedicle	4
Visible base	5
No visible pedicle and base	4

The determination and storage of characteristic point location is carried out by marking them on the image, storage of the information on coordinates of all required points in computer memory and edition on computer monitor. This stage is completed by drawing characteristic distances on the tested image. On the basis of the localization of particular distances, the decision can be taken if the introduced points are located in correct places. The basic criterion is the perpendicularity of relevant pairs of characteristic distances. If there is not, this procedure must be repeated.

Fig. 7 presents the image with correctly marked points and edited distances.

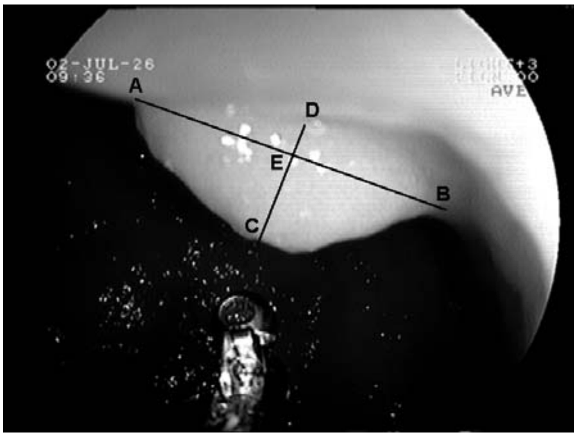

Fig. 7. Points marked on a polyp with no visible pedicle and base [3]

The calculation of distance length is the most important part of the presented algorithm. The distance length required to determine the size of the pathology can be calculated from the following dependence:

$$\overline{P_1P_2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\,, \qquad (7)$$

where: $\overline{P_1P_2}$ - distance length,

x_1, y_1 - P_1 coordinates,

x_2, y_2 - P_2 coordinates.

Figs. 8, 9, 10 present models for tested polyp types and the marked points and distances required for size evaluation.

A. Polyps with no visible pedicle and base

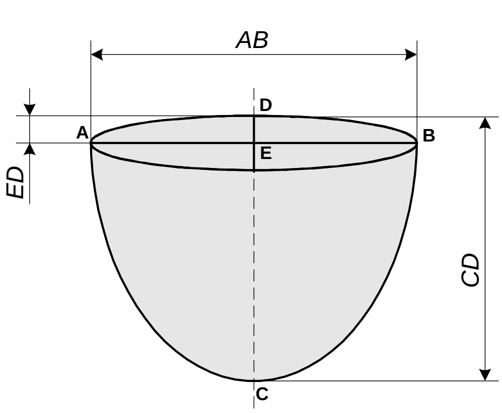

Fig. 8. The points marked on the model of a polyp with no visible base and pedicle

The volume of the pathology can be described by the following dependence:

$$V = \frac{1}{8}\Pi \cdot \overline{AB}^2 \cdot \overline{CE} \cdot \frac{1}{\sqrt{1-\left(\frac{2\overline{ED}}{\overline{AB}}\right)^2}}\,, \qquad (8)$$

gdzie: $\overline{AB}$, $\overline{CE}$, $\overline{ED}$ - characteristic distance lengths.

B. Polyps with the visible pedicle

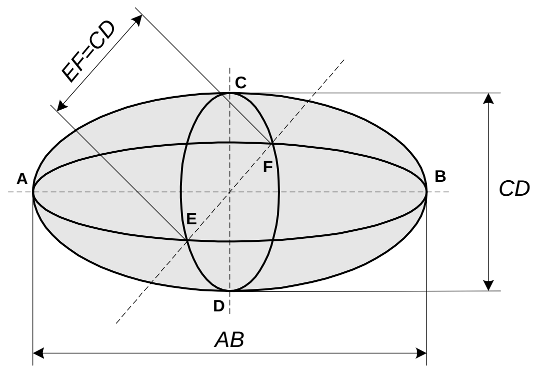

Fig. 9. The points marked on the model of a polyp with the visible pedicle

This volume can be computed from equation:

$$V = \frac{1}{6}\Pi \cdot \overline{AB} \cdot \overline{CD}^2\,, \qquad (9)$$

where: $\overline{AB}$, $\overline{\mathrm{CD}}$ - characteristic distance lengths.

C. Polyps with the visible base

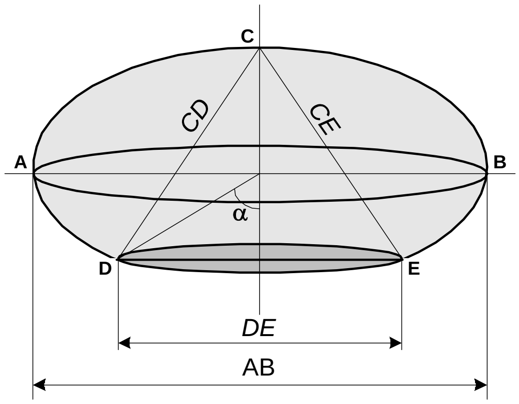

Fig. 10. The points marked on the model of a polyp with the visible base

This volume can be computed from the below dependences:

$$V=\frac{\Pi h}{3}\left(3\sqrt{\left(\frac{\overline{AB}}{2}\sin\alpha\right)^{2}+\left(\sqrt{\overline{CA}^{2}-\left(\frac{\overline{AB}}{2}\right)^{2}}\cos\alpha\right)}-h\right),\quad(10)$$

$$h=\sqrt{\left(\overline{CD}\right)^{2}-\left(\frac{\overline{DE}}{2}\right)^{2}},\quad(11)$$

$$\alpha=arctg\frac{\overline{DE}}{2\left(\sqrt{\left(\overline{CD}\right)^{2}-\left(\frac{\overline{DE}}{2}\right)^{2}}\right)-\left(\sqrt{\left(\overline{CA}\right)^{2}-\left(\frac{\overline{AB}}{2}\right)^{2}}\right)},\quad(12)$$

gdzie: $\overline{AB}$, $\overline{CD}$, $\overline{DE}$ - characteristic distance lengths,
h – polyp height,
α - angles between the ellipse curve and its height.

All lengths are measured in pixels and this causes that volumes are not expressed in SI units but in voxels (the relevance of pixel in 3D space). To recalculate volumes into mm and mm^3 the pathology size must be compared to a size pattern.

IV. Conclusions

The presented mathematical models have made possible to compose the computational algorithm which is the basis for a computer application to determine pathology size in the form of polyps. The proposed method enables independent determinations which helps to take up the most convenient way of treatment.

Further steps in the analysis and model applications are predicted in the determination of errors.

References

[1] Silverstein F.E., Tytgat G.,"Endoskopia przewodu pokarmowego – Atlas", Medycyna Praktyczna Kraków, 1998
[2] Nagasko N., Fujimori T. i in.: "Atlas of Gastroenterologic Endoskop", Igaku-Shoin, Tokyo - New York, 1998
[3] Zdjęcia schorzeń żołądka, Archiwum Instytutu Medycyny Wsi w Lublinie
[4] Moon P., Spencer D.E., "Field theory nandbook – including coordinate systems differential equations and ther solutions", Springer-Verlag. Berlin 1961