Time-Frequency Representation for Non-Stationary Phenomena in Electrical Engineering

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***Abstract* — The paper introduces two-dimensional method of observation and diagnosis of non-stationary signals in electrical engineering. To investigate the methods several experiments were performed using simulated signals. First two-dimensional representations are obtained applying Wigner and Wigner-Ville Distribution . Then local frequency moments are calculated to achieve one-dimensional characteristics. It is shown that such characteristics preserve information about the nonstationarity in point of time, and can be used for calculation of the beginning and duration time of transient states.**

I. INTRODUCTION

REPRESENTATION of signals in time and frequency domain has been of interest in signal processing areas for many years, especially taking in the limelight time-varying non-stationary signals. This kind of representation becomes more and more interested also in electrical engineering. The main motivations which incline to joint time-frequency analysis originate from character of the signals which appear in nowadays power systems and also from constantly increasing requirements for signal processing methods.

Frequency power converters and arc furnaces generate a wide spectrum of harmonic components which deteriorate the quality of the delivered energy, increase the energy losses as well as decrease the reliability of a power systems. Especially large converters systems can be considered as a sources of non-characteristic harmonics and interharmonics. Because of faults the basic component of short circuit current can be distorted by an exponential dc component, and the basic component of voltage by transient oscillating component. In dependence on transmission line parameters and the location or phase of occurring faults, the duration time of discussed transient components could reach value up from 5 to 10 periods of basic components. The estimation of the components parameters is very important for control and protection tasks. The design of harmonics filters relies on the measurement of distortions in both current and voltage waveforms [5,6].

The standard method for study time-varying signals is short-time Fourier transform (STFT) that is based on the assumption that for a short-time basis signal can be considered as stationary. The spectrogram utilizes a short-time window whose length is chosen so that over the length of the window signal is stationary. Then, the Fourier transform of this windowed signal is calculated to obtain the energy distribution along the frequency direction at the time corresponding to the centre of the window. The crucial drawback of this method is that the length of the window is related to the frequency resolution. Increasing the window length leads to improving frequency resolution but it means that the nonstationarities occurring during this interval will be smeared in time and frequency. This inherent relationship between time and frequency resolution becomes more important when one is dealing with signals whose frequency content is changing rapidly [1,2].

A time-frequency characterization of signals that would overcome above drawback became a major goal for signal processing areas. Starting with classical works of Gabor, Ville and Page, there has been an alternative development for study of time-varying spectra. The concept of the Wigner distribution was introduced in the context of quantum mechanics, although reintroduced by Ville for signal analysis. In eighties Claasen and Mecklenbräuker, Janse and Kaizer, Boashash, Rihaczek, Cohen, Choi and Williams developed ideas uniquely situated to the time-frequency situation [7]. Cohen employed characteristic function and operator theory to derive a general class of joint time-frequency representation. It can be shown that many bilinear representations can be written in one general form that is traditionally named Cohen's class [4].

Observing the recent approaches to the time-frequency representations we can separate two main groups in point of the estimation manner as non-parametric and parametric methods. Further, due to different structure of definition equation the non-parametric methods can be parted into groups, which carry out the linear or non-linear operation on the signal. At least if there is a need to scale the time or frequency argument we treat the representations as a scalogram or spectrogram respectively [8,9]

The investigations presented in the paper are scoped at two levels. First level includes calculation of two-dimensional joint time-frequency representations when Wigner and Winger-Ville distribution is applied. Second level concerns calculations of local frequency moment of obtained two-dimensional representation which leads to one-dimensional function of time.

General purpose of the work is to emphasize the advantages and disadvantages of proposed methods in point of their application for time-varying spectral estimation of electrical signals. Some effort to apply local frequency moments of two-dimensional

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representation was also made especially in order to calculate the beginning and duration time of transient states caused by any disturbances.

II. Mathematical Background

This section outlines the character of Wigner and Wigner-Ville distribution including basic comments about the advantages and disadvantages of described approach. The definitions and interpretation of local frequency moments of time-frequency distribution is also introduced.

A. Wigner and Wigner-Ville Distribution

Wigner Distribution (WD) of signal $\mathrm{x}(t)$ is bilinear and non-parametric transformation given by [3,7,8,9]:

$$\mathrm{WD}_x(t,\omega)=\int_{-\infty}^{+\infty}\mathrm{x}\left(t+\frac{\tau}{2}\right)\mathrm{x}^*\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}\omega\tau}\mathrm{d}\tau \quad (1)$$

and could be consider as Fourier transform of instantaneous autocorrelation function. Such approach allowed to observe time-varying spectrum. Bilinear operation on the signal indicates intensity in regions where zero-values are expected. These undesirable components, sometimes called artefacts or cross-terms are usually attributed to the bilinear nature of the distribution and reduce auto-components resolution, obscure the true signal features and make interpretation of the distribution difficult. For multicomponent real signal $\mathrm{y}(t)=\mathrm{x}_1(t)+\mathrm{x}_2(t)$ Wigner Distribution can be expressed by:

$$\begin{aligned}\mathrm{WD}_y(t,\omega)&=\mathrm{WD}_{x_1}(t,\omega)+\mathrm{WD}_{x_2}(t,\omega)+\\&+2\,\mathrm{Re}\{\mathrm{WD}_{x_1,x_1}(t,\omega)\}\end{aligned} \quad (2)$$

It is worth emphasizing that for real signals WD is real function and moreover is an even function of frequency.

First step to reduction of the cross-term components is applying analytic form of the signal which is characterized by zero-value of the spectrum in negative part of frequency axis. It leads to Winger-Ville Distribution (WVD) and allowed to avoid cross-term components which result from interaction between auto-terms hold in positive and negative part of frequency axis, respectively. Introducing then analytic form of signal we can define WVD: [3,7,8,9]:

$$\mathrm{X}_\mathrm{A}(\omega)=\begin{cases}2\mathrm{X}(\omega) & \text{dla } \omega>0\\ \mathrm{X}(0) & \text{dla } \omega=0\\ 0 & \text{dla } \omega<0\end{cases}\Rightarrow \mathrm{WD}_{x_A}(t,\omega)=0 \text{ dla } \omega<0 \quad (3)$$

$$\mathrm{WVD}_x(t,\omega)=\int_{-\infty}^{+\infty}\mathrm{x}_\mathrm{A}\left(t+\frac{\tau}{2}\right)\mathrm{x}_\mathrm{A}^*\left(t-\frac{\tau}{2}\right)\mathrm{e}^{-\mathrm{j}\omega\tau}\mathrm{d}\tau \quad (4)$$

Further reduction of cross-term components is based on the convolution the integrand of Wigner equation with selected kernel function. It leads to Cohen’s generalization of time-frequency distributions [4,8,9].

B. Local Frequency Moments

Two-dimensional effect of the methods, however very valuable when the kind of nonstationarity is characterized, can be inconvenient in point of practical using. Such suggestion leads to calculation of local time and frequency moments of obtained two-dimensional representation. According to the approach one-dimensional characteristics can be achieved with preservation all information about the nonstationarity in time and frequency domain, separately. In this paper authors specially take into the limelight the local frequency moments of Wigner and Wigner-Ville Distributions.

Local frequency moments of Wigner Distribution are determined by considering WD as a function of frequency for fixed time [3,7,8]. Zero order local frequency moment of WD describes equation:

$$\overline{\mathrm{M}^0_{WD_x}}(t)=\int_{-\infty}^{+\infty}\mathrm{WD}_\mathrm{x}(t;\omega)\mathrm{d}\omega=\left|\mathrm{x}(t)\right|^2 \quad (5)$$

and can be interpreted as instantaneous power of signal. The first local frequency moment is than given by:

$$\overline{\mathrm{M}^1_{WD_x}}(t)=\int_{-\infty}^{+\infty}\omega\mathrm{WD}_\mathrm{x}(t;\omega)\mathrm{d}\omega \quad (6)$$

For complex signal $\mathrm{x}(t)=\left|\mathrm{x}(t)\right|\mathrm{e}^{\mathrm{j}\psi(t)}$ Eqn. (6) can be derivate in the form:

$$\overline{\mathrm{M}^1_{WD_x}}(t)=2\pi\psi'(t)\left|\mathrm{x}(t)\right|^2 \quad (7)$$

that consists information about instantaneous frequency of the signal. Unfortunately in case of real signal, when WD is an even function of frequency, $\overline{\mathrm{M}^1_{WD_x}}(t)=0$ and can’t be use as index of nonstationarity.

The idea introduced in the paper is to calculate the local frequency moment of Winger-Ville distribution with lower bound of integration range equal to zero. Proposed approach, especially considering $\mathrm{WVD}_x(t,\omega)=0$ for $\omega<0$, preserves information about the nonstationarity in point of time. Calculated moments could be further normalized which leads to definitions of applied equations in form:

- zero order local frequency moment of WVD:

$$\overline{\mathrm{M}^0_{WVD_x}}(t)=\int_{0}^{+\infty}\omega\mathrm{WVD}_\mathrm{x}(t;\omega)\mathrm{d}\omega \quad (8)$$

- first order local frequency moment of WVD:

$$\overline{\mathrm{M}^1_{WVD_x}}(t)=\int_{0}^{+\infty}\omega\mathrm{WVD}_\mathrm{x}(t;\omega)\mathrm{d}\omega \quad (9)$$

- normalized first order local frequency moment of WVD:

$$\overline{\Omega^{1}_{WVD_x}}(t)=\frac{\int_0^{+\infty}\omega \mathrm{WVD_x}(t;\omega)\mathrm{d}\omega}{\int_0^{+\infty}\mathrm{WVD_x}(t;\omega)\mathrm{d}\omega} \tag{10}$$

Eqn. (10) can be then interpreted as a function which describes position of central point of the instantaneous spectrum for fixed time. It is worth emphasizing that following this interpretation, information about the frequency structure is lost. However information about the transient events is still preserved. The crucial significance of characteristic $\overline{\Omega^{1}_{WVD_x}}(t)$ is then the opportunity to apply it for detection of nonstationarity or duration time of transient states.

III. EXAMPLES AND RESULTS

The character of Wigner-Ville distribution and its normalized local frequency moments has been tested for two simulated signals: sum of two cosine functions and transient signal in RLC branch. Sampling frequency equals 5kHz. Describing the local frequency moment we have to notice that cross-terms also take a part in the calculations. This influence manifest itself in oscillations around the true value of central point of spectrum which is achieved only when cross-term has zero-value. To avoid mentioned influence the authors propose to use median filter to suppress the oscillations. When small order of the filter is used no influence on dynamic of curve is achieved.

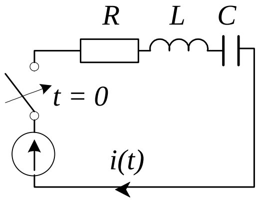
A. Sum of cosine functions

First signal, illustrated in Fig. 2a, can be described by:

$$\begin{aligned}\mathrm{x}(t)&=10\cos(100\pi t)[1(t)-1(t-0.2)]+\\&+5\cos(500\pi t)[1(t-0.1)-1(t-0.2)]\end{aligned} \tag{11}$$

According to bilinear nature of the transformation, one cross-term component 150Hz appear between auto-terms 50Hz and 250Hz (Fig. 2b). Normalized local frequency moment smoothed with median filter, order 50, was illustrated in Fig. 2c. We can clearly detect appearing time of nonstationarity. Observing displacement the direction of the curve forward higher frequencies allow us to comment that higher components have appeared in the signals. However no details about number of the components and its energy participation can be described. Very interesting effect brings normalization process of first order local frequency moment of WD by instantaneous power of the signal which leads directly to derivative of the phase. In case of real signals it allows to determine moments when signal obtains zero value (Fig.3).

B. Transient state in RLC branch



$e(t)=230\sqrt{2}\sin(\omega t-90^o)V$

$R=4\Omega,\ L=0.1\mathrm{H},\ \mathrm{C}{=}10\mu\mathrm{F}$

$\Rightarrow \mathrm{R}{<}2\sqrt{\frac{\mathrm{L}}{\mathrm{C}}},\ \varphi\approx\psi_e=-90^o$

Fig. 1. Example B – analyzed circuit and its parameters.

From analysis of above circuits we can describe signal i(t) as:

$$i(t)\approx\frac{230\sqrt{2}}{287}\left(\sin(100\pi t)-\frac{1000}{100\pi}e^{-20t}\sin(1000t)\right)1(t) \tag{12}$$

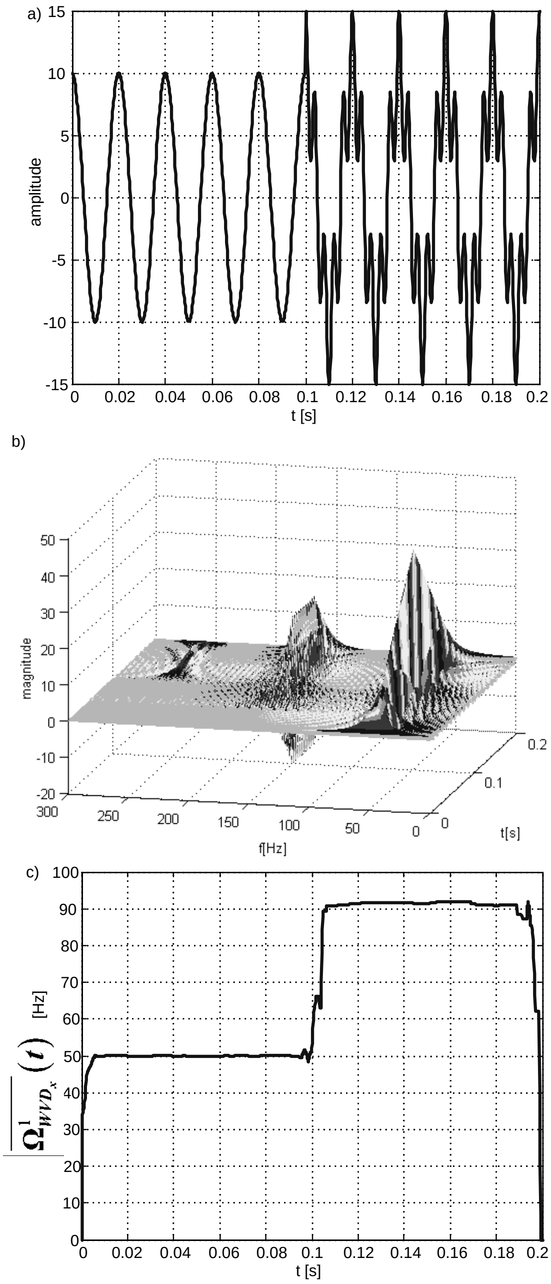


Fig. 2. Example A – sum of cosine functions (a), its Wigner-Ville distribution (b) and normalized local frequency moment smoothed with median filter (c).

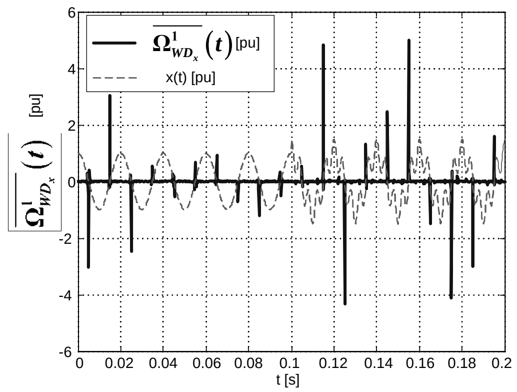


Fig. 3. Normalized local frequency moment of Wigner Distribution as a detector of zero-values of the signal from Example A.

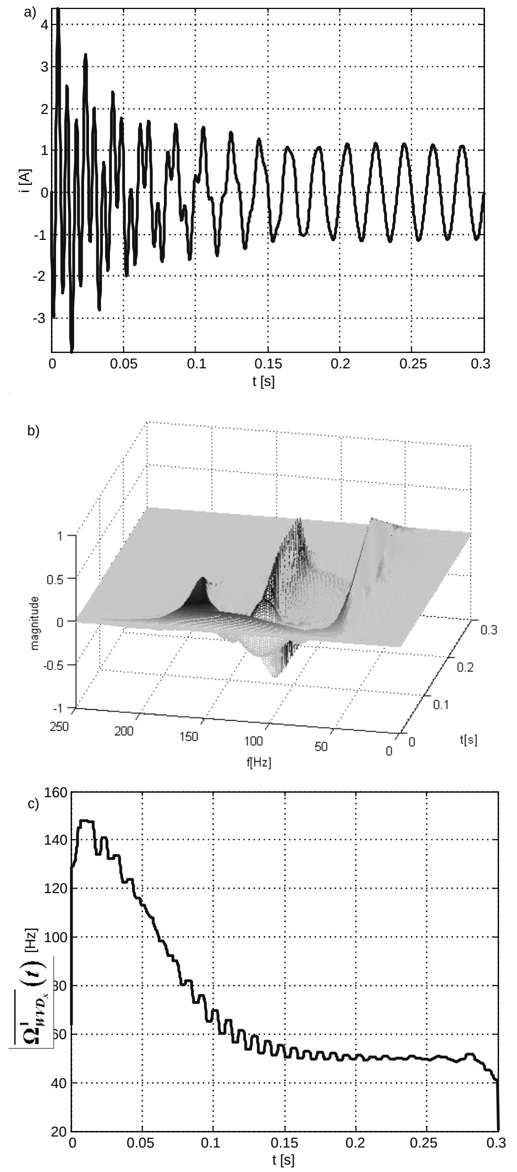


Fig. 4. Example B – current signal in RLC branch under transient conditions (a), its Wigner-Ville distribution (b) and normalized local frequency moment smoothed with median filter (c).

Waveform of signal i(t) and its Wigner-Ville distribution was illustrated in Fig. 4a-4b. Transient state manifest itself in existing the transient component about 160Hz and input component 50Hz. Observing Fig. 4c we can recognize shifting the position of central point of the instantaneous spectrum forwards input components as a results of decaying transient component. The duration time of the transient state can be also clearly characterized. Similarly as in Example A detailed information about the frequency or amplitude of the transient component is hidden.

IV. CONCLUSIONS

It has been shown that the non-parametric time-frequency representation such as Wigner and Wigner-Ville distributions, can be used for parameter estimation of distorted, non-stationary signals. Discussed methods are computationally complex and very often obscured by the cross-terms influences, but improving frequency concentration effect is achieved in comparison to Fourier algorithm.

Two-dimensional effect of the methods, however very valuable when the kind of nonstationarity is described, can be inconvenient in point of practical using. Therefore further investigations include calculation of local frequency moment which are one-dimensional function of time. Normalized first order local frequency moment of WD leads directly to derivative of the phase and in case of real signals allows to determine moments when signal obtains zero value. Normalized first order local frequency moment of WVD illustrates how the centre point of spectrum changes in time. Although following that interpretation all details about the frequency components and its energy participation is lost, information about the time parameters of nonstationarity is clearly preserved. It gives idea of applying the discussed approach for detection and classification.

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